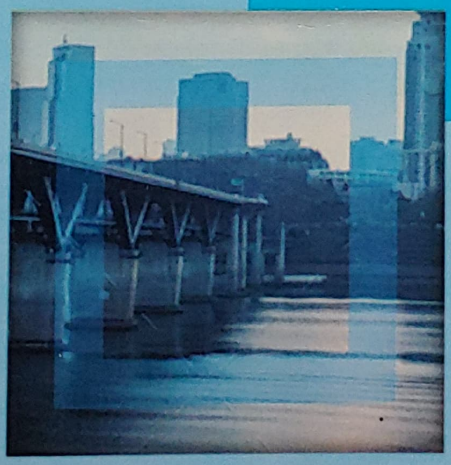
1st Edition 2020

A Handbook of

Reinforced Concrete Structures

Part - I

- Chapterwise Theory analysis
- 12+ Basic Concepts
- 20+ Short Techniques
- 40+ Problem Solutions
- QB Solution (2013-2019)



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A Handbook of
Reinforced Concrete Structures
(Part-I)

[Especially for the students of Civil Engineering, BECM and URP department]

Written By

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A Handbook of Reinforced Concrete Structures (Part-I)

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Most Deeply Dedicated to
My Beloved Parents

[The sacrifices and contributions of every parents
for their child are immense and undeniable]

"O Allah! Bestow on my parents The Mercy even as they cherished me in childhood"
[The Holy Quran; 17:24]

Preface

First of all, the book is written for those students who love to study neatly. In fact, in the beginning there was no intention to write a book like this approach. The author studied on Reinforced Concrete Structures during third year of undergraduate in civil engineering. However, there were many books on this subject but there were no well-arranged books which was easily fathomed. So the author personally combines different textbooks as well as notes the theories and problem solutions under the chapters according to the class lecture. That is why the book is written during fourth year of undergraduate of the author to save the valuable notes from being lost inexpertly as well as for the students who are thirst for knowledge. The book is arranged stepwise with chapter-based theory analysis, basic concepts, problem solving techniques and examination useful questions. It is better to say that the problems on the both design and analysis of concrete structures are solved in the both practicing method of WSD and USD. The author has tried to make it students friendly by providing neat sketches and illustrations with practical problems, wherever necessary. Hopefully the book can be useful to those-

- who wants to pass in the examination in a short time.
- who wants to get a letter mark easily.
- for all civil engineering students.
- for all civil engineering job seeker.

The author is very optimist that this book will be received with open arms by the students and educators. As well as this book will motivate the learners to be more devoted to the subject.

In this work as a human, it is possible that errors may occur. Any comments and suggestions about the book are always welcome. There will be a request to the users, if there is any mistakes and omissions, it will be thankfully acknowledged. The author will be glad to consider them for inclusion in the next edition of the book. Insha-Allah.

Just write to the author via mail.

Chandpur, Bangladesh
5 October, 2020

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Acknowledgement

At the beginning, I would like to thank and my immense gratefulness to The Almighty ALLAH for his kindness and grace to make me capable to compose the handbook. The handbook is the compilation and composed of valuable class lectures of a semester long delivered by the honorable teachers Prof. Dr. Abu Zakir Morshed and Associate Prof. Dr. Md. Mahfuzur Rahman of KUET, engineering university. As well as combining the different textbooks and semester hand notes, the book has been transformed into a complete easy and smooth handbook.

Finally I would like to say that the gratitude of my parents behind my coming so far is undeniable. As well as I sincerely thank my friends of the civil engineering department whose help me much in every steps of my study journey.

Especially I am so grateful to my close friend Rafsan Noon for his outstanding cover design. Also I want to remember my another friends Fairooz Adnan and Zahed Hossain for their immediate help. With that I also would like to give endless thanks to my elder brother Jamir Uddin Mamun as well as my close friends Rakib Hasan and Ahnaf Hasan for giving me advice and encourage to complete the work.

In addition, the author is very grateful to the writers of all the textbooks cited in reference section for the acquiring basic knowledge, these helped the author to make an easy handbook.

Chandpur, Bangladesh
5 October, 2020

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Chapter-1
Introduction to Reinforced Concrete

1.1 Theory Analysis:

Q-1. Define reinforced cement concrete? What are the advantages and disadvantages of reinforced concrete?

Ans: ***Reinforced cement concrete***: The concrete has a high compressive strength and a very low tensile strength wherein the steel reinforcement provides the tensile strength lacking in the concrete. This working combination of concrete and steel is known as the reinforced cement concrete (R.C.C).

Advantages of reinforced concrete:

(i) Reinforced concrete structures are very rigid.
(ii) It has a relatively high compressive strength.
(iii) It has better resistance to fire than steel.
(iv) It has a long service life with low maintenance cost.
(v) It is the most economical structural material for dam, piers, footings & floor slabs.

Disadvantages of reinforced concrete:

(i) It has a low tensile strength of about one-tenth of its compressive strength.
(ii) It has a low compressive strength as compared to steel.
(iii) The low strength per unit of weight of concrete leads to heavy & large members.
(iv) The placing & curing of concrete is not carefully controlled as of other materials.
(v) Crack develops in concrete due to shrinkage & creep.

Q-2. What are the different types of load introduced on concrete structures?

Ans: There are different types of load that is introduced on concrete structures such as-

(i) Dead load
(ii) Live load
(iii) Environmental load
    (a) Snow load
    (b) Wind load
    (c) Earthquake load

Q-3. Discuss the fundamental assumptions (propositions) for the reinforced concrete behavior. [***CE-2014, 2013; BECM-2019, 2018, 2017***]

Ans: The fundamental assumptions for reinforced concrete behavior are as follows:

(i) The internal forces (SF, BM, and Stress) at any section of a member are in equilibrium with the effects of the external loads at the section.

(ii) The strain in reinforcing bar is same as the surrounding concrete ($\epsilon_s = \epsilon_c$).
(iii) Cross section remains plane after & before loading in the member.
(iv) The tensile strength of concrete is only a small fraction of its compressive strength. So it is assumed that concrete is not capable of resisting any tensile stress.
(v) The theory is based on the actual stress-strain relationship and strength properties of the two constituent materials.

Q-4. Describe the scope of using high strength concrete. [*CE-2015*]
Ans: High strength concrete refers to concrete having uniaxial compressive strength in the range of about 8,000 to 20,000 psi or higher.
The scope of using high strength concrete are described below:
(i) Columns of tall concrete building
(ii) Offshore oil structures
(iii) Parking garages
(iv) Dam spillways
(v) Bridge deck overlays
(vi) Warehouse
(vii) Heavy industrial slab
Advantages of using high strength concrete:
(i) It saves valuable floor space.
(ii) It helps in choosing the limited small concrete sections.
(iii) It has more shear capacity than normal concrete.
(iii) It provides good safety and serviceability.

Q-5. Why steel is widely used as a reinforcing material in reinforced concrete?
Ans: Steel is widely used in reinforced concrete because:
(i) It possesses high tensile strength & elasticity.
(ii) Its strength is sufficient both for tension and compression.
(iii) It develops perfect bonding with concrete.
(iv) Its thermal expansion coefficient is nearly to that of concrete.
(v) It is durable and easy to cut & bent.
(vi) It reduces X-sectional dimensions of compression members.
(vii) It provides relatively large chemical adhesion.

Q-6. Why deformed bar instead of plain bar is suggested in RCC structures?
Ans: The reasons are described below:-
(i) Increasing resistance to slip between reinforcement bar and concrete.
(ii) Ensuring sufficiently strong bond between steel bar and concrete.

(iii) Providing a high degree of interlocking with between steel bars.
(iv) Giving the closely spaced rib-shaped surface.

Q-7. Discuss the modulus of elasticity of concrete.
Ans: The *modulus of elasticity,* E_c is the slope of the initial straight portion of the stress-strain curve for the concrete.
Concrete has no clear-cut modulus of elasticity. Its value varies with different concrete strengths, concrete age, type of loading, and the characteristics and proportions of the cement and aggregates.
The *modulus of elasticity,* E_c is given by the empirical equations as follows:
(a) In FPS units, $E_c = 33{w_c}^{1.5}\sqrt{f_c'}$
For normal-weight concrete weighing 145~150 lb/ft^3, $E_c = 57000\sqrt{f_c'}$
(b) In SI units, $E_c = 4730\sqrt{f_c'}$

Q-8. Discuss the design philosophy between steel and concrete from their stress-strain diagram. Also describe their consequences.
Ans: Design philosophy between steel and concrete:-

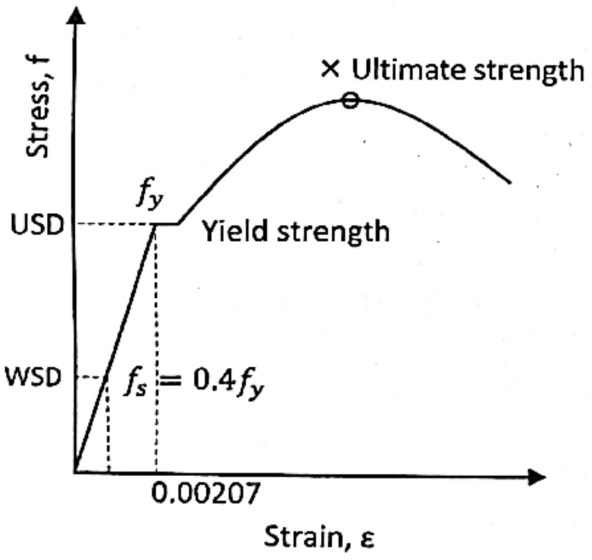


Fig: Steel bar stress-strain diagram

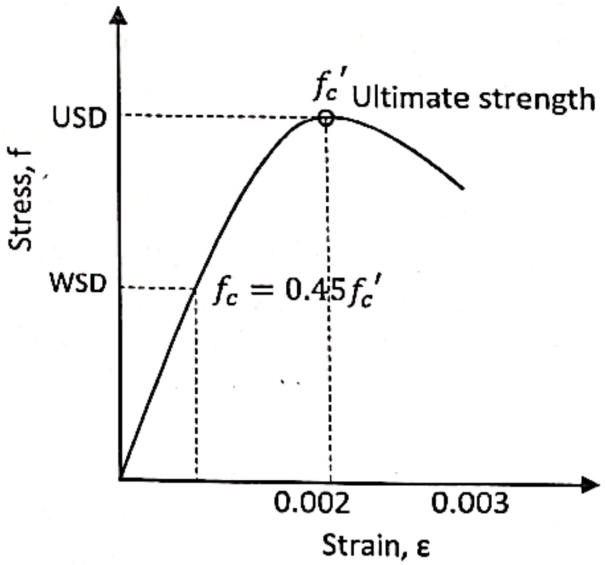


Fig: Concrete stress-strain diagram

From the concrete stress-strain diagram, it can be seen that curve consist of an initial relatively straight elastic portion up to $0.45f_c'$ in which stress and strain are closely

proportional. Concretes reach their ultimate strength at strain of about 0.002. Concrete does not have a definite yield strength. The failure of concrete occurs at a strain of about 0.003 (ACI 10.2.3).

By the fundamental assumption of reinforced concrete, the yield strength of steel bar, f_y is considered corresponding to the strain of 0.00207 that is nearly close to strain of 0.002 of concrete. So, ultimate strength of concrete f_c' and yield strength (not ultimate strength) of steel bar, f_y are applied in *ultimate strength design* (USD) method. Hence working strength f_c for concrete and f_s for reinforcing steel bar are applied in *working stress design* (WSD) method. Anyway the stress condition for both the steel and concrete follow Hook's law, $f \propto \in$ in WSD metod.

Consequences of design philosophies:-

(i) In USD method, load is factored increasingly so it is needed less materials.

(i) In WSD method, strength of materials is considered lower value so it is needed more materials and cost may be high.

(iii) ACI used the WSD method about till 20 years ago.

(iv) But BNBC uses both the WSD and USD method still now.

Q-9. Discuss the methods of design of RCC structures.

Ans: There are three available methods of design of RCC structures:-

1. *Working stress method:* This method is also known as *modular ratio* or *elastic method.* In this method, the actual values of forces & moments acting on a structure are considered but the stresses in concrete & steel bar are restricted to only a fraction of their true strengths by a factor of safety of 0.45 in concrete and 0.40 in steel.

2. *Limit state method:* This method is also known as *plastic method.* In this method, the structure is designed to withstand safely all loads liable to act on it throughout its life. It should also satisfy the serviceability requirements such as prevention of excessive deflection, excessive cracking and excessive vibrations. The acceptable limit for the safety and serviceability requirements before failure occurs is called *limit state.*

3. *Ultimate strength method:* This method is also known as *load factor method.* In this method of design, the true margin of safety is accurately known. The ultimate strength procedure is based upon the results obtained from the experimental investigations depicting the true behavior of the structures.

Q-10. Distinguish between WSD and USD method.

Ans: The differences between WSD and USD method are given below:

WSD	*USD*
i) Consider at comparatively low stress, hence the stress & strain have a linear relationship.	i) Consider at comparatively high stress, hence the stress & strain have no linear relationship.
ii) DL & LL are not multiplied by increasing factor.	ii) DL & LL are multiplied by some increasing factor.
iii) Strength reduction factor (φ) is not used.	iii) Strength reduction factor (φ) is used.
iv) Maximum concrete stresses is up to elastic limit.	iv) Ultimate strength of concrete is considered.
v) Actual test strength of the two materials are used in the design of members.	v) Modular ratio (n) is used in the design of members.
vi) Stability of structure is less than USD.	vi) Stability of structure is more than USD.
vii) High cost design method.	vii) Low cost design method.

1.2. Basic Concepts:

1. Modulus of Elasticity of concrete (E_c):

(a) In FPS units, $E_c = 33{w_c}^{1.5}\sqrt{f_c'} \quad (psi)$

For normal-weight concrete weighing $w_c = 145\sim150\ lb/ft^3$,

$$or,\ E_c = 57000\sqrt{f_c'} \quad (psi)$$

(b) In SI units, $\boldsymbol{E_c = 4730\sqrt{f_c'} \quad (MPa)}$

2. Modulus of Elasticity of steel (E_s):

(a) In FPS units, $E_s = 29 \times 10^6 \ \ psi$

(b) In SI units, $\boldsymbol{E_s = 200\ GPa = 2 \times 10^5\ \ MPa}$

3. Modular Ratio (n):

Modular Ratio, $n = \dfrac{E_s}{E_c}$

1.3. Problem Solution:

Pb-1. If a concrete weighing 140 lb/ft^3 and having concrete stress 3500 psi, what is the modulus of elasticity of the concrete?

Sol^n:
We know, the modulus of elasticity of the concrete is,

$$E_c = 33{w_c}^{1.5}\sqrt{f_c'} = 33 \times (140)^{1.5}\sqrt{3500} = 3.234 \times 10^6\ psi$$

Pb-2. For a normal-weight concrete having a stress of 4000 psi, compute the modulus of elasticity of the concrete?

Sol^n:
We know, the modulus of elasticity of the concrete is,

$$E_c = 57000\sqrt{f_c'} = 57000\sqrt{4000} = 3.605 \times 10^6\ psi$$

Pb-3. The concrete stress of a normal-weight concrete is of 35 MPa. What is the modulus of elasticity of the concrete?

Sol^n:
We know, the modulus of elasticity of the concrete is,

$$\boldsymbol{E_c = 4730\sqrt{f_c'} = 4730\sqrt{35} = 27.98 \times 10^3\ MPa}$$

Pb-4. If a concrete having stress of 3500 psi and the modulus of elasticity of the concrete is of $3.5 \times 10^6\ psi$, calculate the weighing of concrete.

Sol^n:
We know, the modulus of elasticity of the concrete is,

$$E_c = 33{w_c}^{1.5}\sqrt{f_c'}$$

$$\Rightarrow 3.5 \times 10^6 = 33{w_c}^{1.5}\sqrt{3500}$$

$$\Rightarrow {w_c}^{1.5} = 1792.75$$

$$\therefore {w_c}^{1.5} = 148\ lb/ft^3$$

Pb-5. If the concrete stress of a normal-weight concrete is of 4000 psi and modulus of elasticity of steel is of $29 \times 10^6\ psi$, what is the modular ratio?

Sol^n:

$$Here,\ E_c = 57000\sqrt{f_c'}$$

We know, the modular ratio is,

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \approx 8$$

Pb-6. The concrete stress of a normal-weight concrete is of 30 MPa and the modulus of elasticity of steel is of $200\ GPa$. Calculate the modular ratio?

Sol^n:

$$Here,\ E_c = 4730\sqrt{f_c'}$$

We know, the modular ratio is,

$$n = \frac{E_s}{E_c} = \frac{200 \times 10^3}{4730\sqrt{30}} = 7.72 \approx 7.50$$

Chapter-2
Analysis & Design of Beams

2.1. Theory Analysis:

Q-1. Define beam. What are the functions of beam in structure?
Ans: ***Beam:*** A beam is a structural horizontal member that carries only the vertical loads as well as which is capable of resisting the bending moment.
The functions of beam in structure are as follows:
i) It can carry the vertical gravitational forces.
ii) It can be used to carry horizontal loads like as earthquake or wind load.
iii) The loads carried by the beam are transferred to columns, walls or girders.

Q-2. Discuss the behavior of reinforced concrete beam under increasing load. *[CE-2019, 2014]*
Ans: The behavior of reinforced concrete beam under increasing load is described below:

i) Stresses elastic and section uncracked: Initially the tensile stress in the concrete is smaller than the modulus of rupture, so that no tension cracks develop. The entire concrete is effective in resisting stress in compression on one side and in tensile on the other side of the neutral axis.

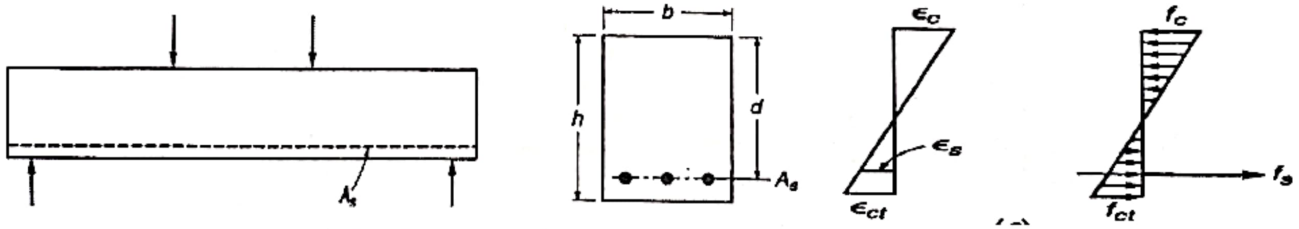


Fig: Stresses elastic and section uncracked

ii) Stresses elastic and section cracked: When the load is more increased, the tensile strength of concrete is soon reached yield limit and at this situation tension cracks develop. In a cracked section, the concrete does not transmit any tensile stresses and the steel is called upon to resist the entire tensile stress.

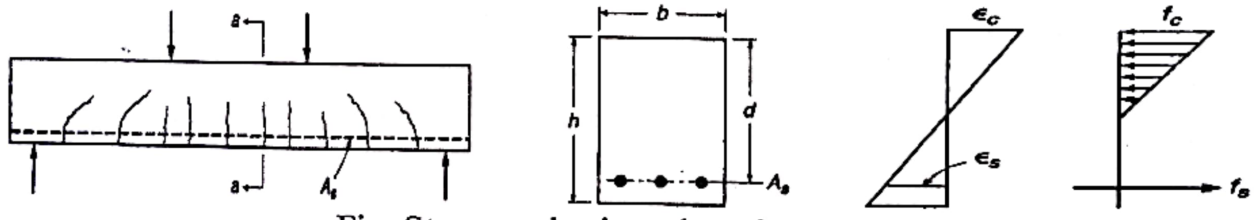


Fig: Stresses elastic and section cracked

iii) Stresses inelastic and section cracked: When the load is still more increased, stresses and strains rise corresponding and are no longer proportional. It causes a invariable shape as like as parabolic, trapezoidal or otherwise.

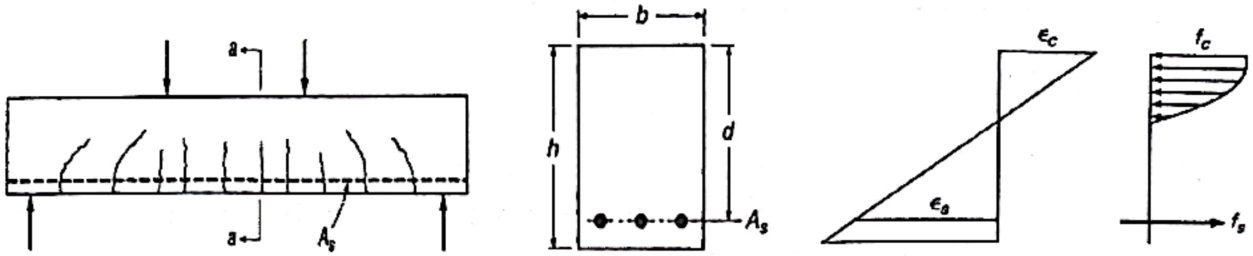
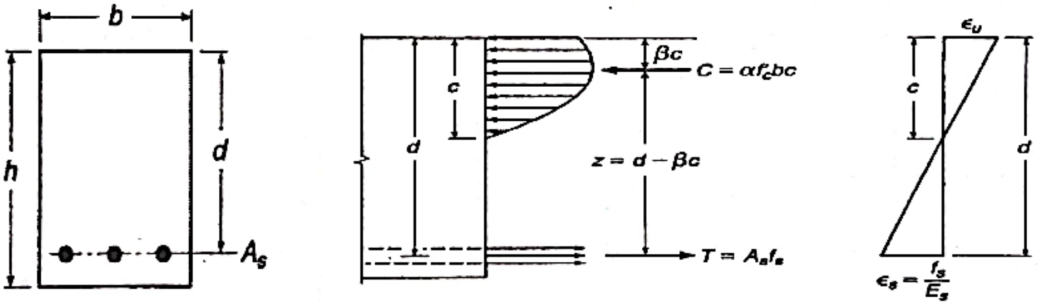


Fig: Stresses inelastic and section cracked

Q-3. Derive the expression for calculating nominal moment at which the beam will fail. [*CE-2018*]
or, Derive the flexural design equation for a singly reinforced beam by USD method. [*CE-2015*]

Ans: The maximum moment can be calculated from the laws of equilibrium and from the assumption that plane cross section remains plane.



If the plane section of beam is in equilibrium, then

$C = T$

$\Rightarrow \alpha f_c' bc = A_s f_s$

$\Rightarrow c = \frac{A_s f_s}{\alpha f_c' b} = \frac{\rho b d f_s}{\alpha f_c' b}$ $\quad [\because A_s = \rho b d]$

$\therefore c = \frac{\rho d f_y}{\alpha f_c'}$ $\quad [At\ failure, f_s = f_y]$

Moment, $M = Tz = A_s f_s(d - \beta c)$.....(1)

$M = Cz = \alpha f_c' bc(d - \beta c)$.....(2)

From eqn. (1), Nominal Moment,

$$M_n = A_s f_s \left(d - \beta * \frac{\rho d f_y}{\alpha f_c'}\right)$$

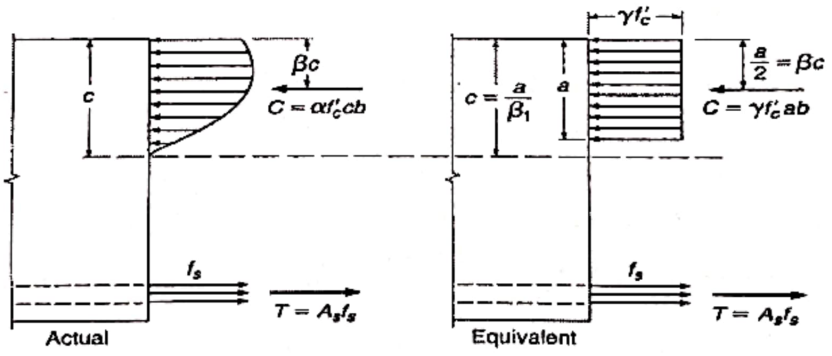
$$\Rightarrow M_n = \rho b d f_y \times d\left(1 - \frac{\beta}{\alpha} \times \frac{\rho f_y}{f_c'}\right); \qquad \left[\therefore \frac{\beta}{\alpha} = \frac{0.425}{0.72} = 0.59\right]$$

$$\therefore M_n = \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

[Derived]

Q-4. Derive the steel ratio formula for a beam under balanced condition. [***CE-2015, 2013***]

Ans: For equivalent rectangular stress distribution we get,



Now, Concrete compression force at failure is,

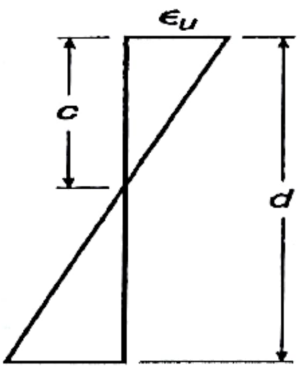
$C = \alpha f_c' cb = \gamma f_c' ab$ $\qquad \left[\gamma = \alpha \frac{c}{a}\right]$

$\Rightarrow C = \gamma f_c' b \beta_1 c$ $\qquad [a = \beta_1 c, from\ figure]$

$\therefore C = 0.85 f_c' b \beta_1 c$ $\qquad [\gamma = 0.85\ for\ f_c' \leq 4000\ psi\ ]$

And, Steel tensile force, $T = A_S f_s$

From similar triangle,



$$\frac{c}{\epsilon_u} = \frac{d-c}{\epsilon_y}$$

$$\Rightarrow \frac{\epsilon_y}{\epsilon_u} = \frac{d-c}{c}$$

$$\Rightarrow \frac{\epsilon_y + \epsilon_u}{\epsilon_u} = \frac{d-c+c}{c}$$

$$\Rightarrow \frac{\epsilon_y + \epsilon_u}{\epsilon_u} = \frac{d}{c}$$

$$\therefore c = \frac{\epsilon_u}{\epsilon_u + \epsilon_y} d$$

Then from equilibrium condition, $T = C$

$\Rightarrow A_s f_s = 0.85 f_c' b \beta_1 c$

$\Rightarrow \rho_b f_y b d = 0.85 f_c' b \beta_1 \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y} d$ $\qquad [A_s = \rho b d \; ; \; At \; failure, f_s = f_y]$

$$\therefore \rho_b = 0.85 \, \beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$$

[Derived]

Q-5. What is Whitney's stress block? Explain why is it used in ultimate strength design (USD) method? [***Private University***]

Ans: ***Whitney's stress block***: Whitney replaced the actual curve stress block with an equivalent rectangular stress block of intensity $0.85 f_c'$ from only f_c'. This stress block is known as Whitney's stress block.

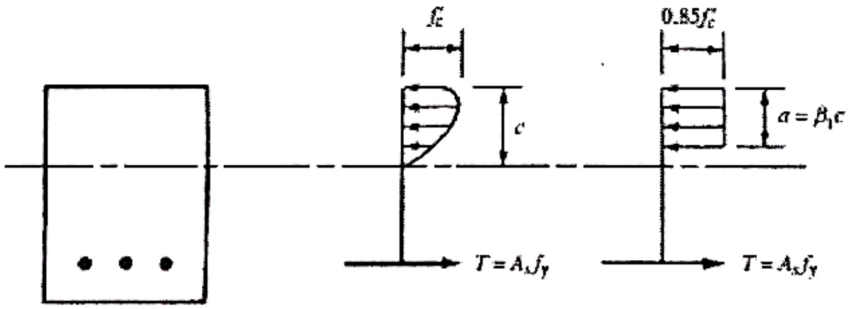


Fig: Actual and equivalent stress distributions at failure

Evidently, both the actual and equivalent stress distributions result in the same total compression force C applied at the same location as in the actual member when it is on the point of ultimate or failure. As well as the concrete is assumed to crush at a strain of about 0.003 at the ultimate strength of f_c'. So, Whitney's stress block is used in ultimate strength design (USD) method.

Q-6. What do you mean by under reinforced, over reinforced and balanced design? What type of design is preferable and why? [*CE- 2014, 2013, 2012; BECM-2018, 2017, 2016*]

Ans: ***Under Reinforced Design:*** When the amount of steel provided is less than the amount of steel required for a balanced section, then the design is called under reinforced design.

Over Reinforced Design: When the amount of steel provided is greater than the amount of steel required for a balanced section, then the design is called over reinforced design.

Balanced Design: When the stress acting in concrete and that applied in steel section, they will attain permissible strain value simultaneously for a balanced section, then the design is called balanced design.

Preferable design: Brittle materials does not warn before failure. Hence failure of ductile material is most preferable that is in case of reinforced concrete, failure of steel is preferable. On other words, under reinforced design is preferable.

Q-7. What is singly & doubly reinforced beam? When doubly reinforced beam is used? [*CE-2017; BECM-2017*]

Ans: ***Singly reinforced beam:*** As tensile strength of concrete is very low as compared to its compressive strength, so the beam which is longitudinally reinforced by the steel only in tension zone, it is known as singly reinforced beam.

Doubly reinforced beam: When the moment capacity of a beam needs to be increased and beam cross-section is also limited, then the reinforcement is provided by the steel in both tension and compression zone of the beam, it is known as doubly reinforced beam.

Doubly reinforced beam is used---

i) If the beam depth is restricted and it requires to increase the moment of resistance.
ii) When the beam is continuous over several supports.
iii) The member is subjected to the impact force.
iv) The member is subjected to eccentric loading.

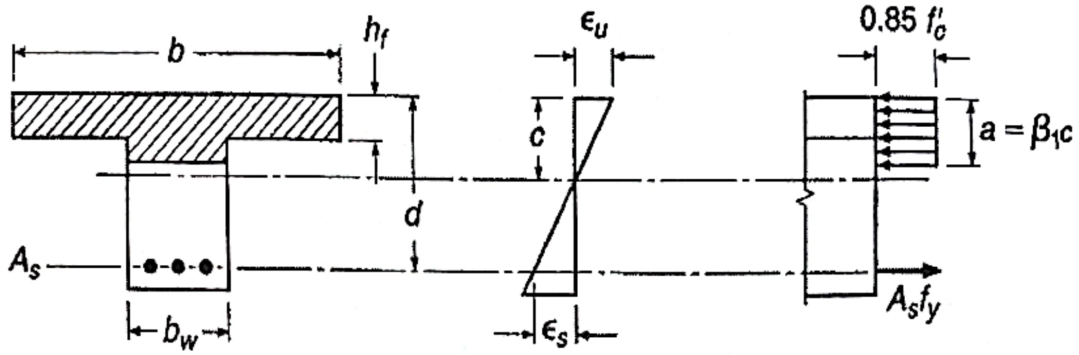
Q-8. What are the advantages of using compression reinforcement in RC beams? [*CE- 2018; BECM-2016*]

Ans: Generally reinforcement is provided in the tension zone in a singly reinforced beam. But in doubly reinforced beam, using extra amount of reinforcement as compression

reinforcement it increases the moment capacity as well as helps to restrict the RC beam section in limit for architectural reasons.

Q-9. Draw strain and equivalent stress distribution for T-beam with possible positions of neutral axis. [***CE- 2019, 2018***]

Ans: The strain and equivalent stress distribution for T-beam is shown below:-



Q-10. When a beam is considered as a T-beam? How can you calculate the effective flange width of a T-beam? [***CE- 2015; BECM-2019***]

Ans: ***T-beam:*** When beams and slabs are monolithically cast; beam stirrups and bent bars extend up into the slab, a part of the slab will act with the upper part of the beam to resist longitudinal compression. The resulting beam cross section is T shaped and evidently called T beams.

The effective flange width of a T-beam:

1. For symmetric T beams:
   Effective flange width,

$$b = \begin{cases} i)\ 16h_f + b_w \\ ii)\ \dfrac{Span\ length\ of\ the\ beam}{4} \\ iii)\ Centerline\ beam\ spacing \end{cases} ; which\ is\ minimum$$

2. For beams having a slab on one side only:
   Effective flange width,

$$b = \begin{cases} i)\ 12h_f + b_w \\ ii)\ \dfrac{Span\ length\ of\ the\ beam}{12} \\ iii)\ \dfrac{Centerline\ beam\ spacing}{2} + b_w \end{cases} ; which\ is\ minimum$$

Q-11. What is True and False T-beam? [***BECM-2019***]

Ans: ***True T-beam:*** When the neutral axis of a T-beam is in the web, it is called True/Real T-beam. *It can be analyzed by the using of formula of T-beam design.*

False T-beam: When the depth of neutral axis is less than or equal to the flange thickness h_f of a T-beam, it is called False T-beam.

It can be analyzed by the using of formula of singly reinforced beam design.

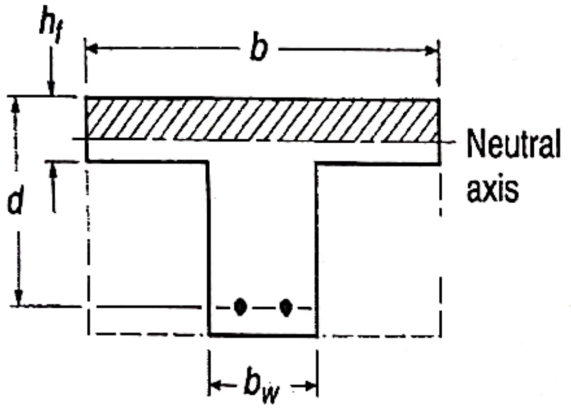


Fig: False T-beam

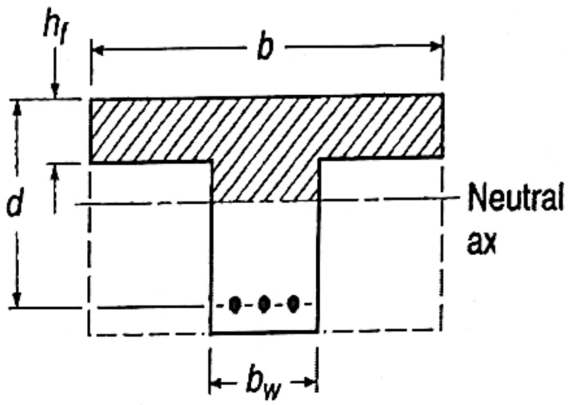


Fig: True T-beam

Q-12. What is meant by effective flange width? Why effective flange width is considered instead of full flange width in design of T-beam? [***CE-2011***]

Ans: ***Effective flange width***: The flange width which may be smaller than the actual flange width but is considered to be uniformly stressed at the maximum value, is called effective flange width.

If the flange is little wider than the stem width, the entire flange can be considered effective in resisting compression. Elements of the flange midway between the beam stems are less highly stressed in longitudinal compression than those of elements directly over the stem. For these reasons, effective flange width is considered instead of full flange width in design of T-beam.

Q-13. What is the advantage of using T-beam? [***CE- 2016***]

Ans: The advantages of using T-beam are as follows:

i) It is built with the slab and beam monolithically cast.

ii) It makes the construction work comparatively economical.

iii) It keeps the beam section smaller by making the slab as a compression resistance to reduce dead load.

Basic Questions:

Q-14. Define transformed, cracked and uncracked section.

Ans: ***Transformed section:*** The section in which the actual area of the steel (A_s) is replaced with an equivalent concrete area equal to nA_s located at the level of steel is called transformed section. Here, n = modular ratio.

Cracked section: The section in which reinforced-concrete members are under service conditions and in the tension zone is known as cracked section.

Uncracked section: The section in which the tensile stress in the concrete is smaller than the ultimate tensile strength of the concrete and there will be no cracking occurring is known as uncracked section.

Q-15. What is singly reinforced section and doubly reinforced section?

Ans: ***Singly reinforced section:*** The section in which steel reinforcements are provided only on tension zone of RC flexural member is known as singly reinforced section.

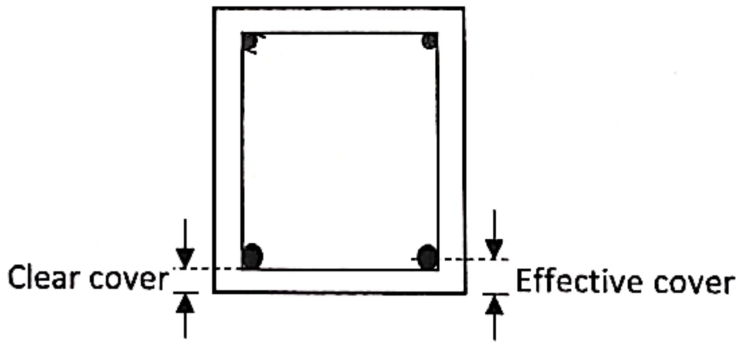
Doubly reinforced section: The section in which steel reinforcements are provided on both tension and compression zone of RC flexural member is known as doubly reinforced section.

Q-16. Define clear cover and effective cover.

Ans: ***Clear cover:*** The distance between the bottom of the bars and the most bottom edge (concrete face) of the beam is called clear cover (cc).

Effective cover: The distance between the center of the reinforcement bar and the most bottom edge (concrete face) of the beam is called effective cover (ec).

$$\therefore Effective\ cover = clear\ cover + \frac{Bar\ diameter}{2}$$



2.2. Basic Concepts:

1. Total depth of beam:

Support condition	Total depth, h
Simply supported	$\frac{L}{12} \sim \frac{L}{16}$
One end continuous	$\frac{L}{18}$
Both ends continuous	$\frac{L}{21}$
Cantilever	$\frac{L}{8}$

*L = span length of a beam (in.)

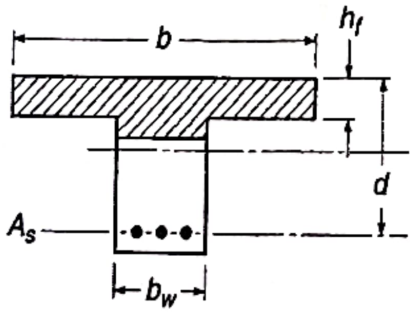
Or,

Thumb rule: Total depth of beam in inch is equal to the span of beam in feet. ∴ h (in.) = L (ft)
Beam depth (in.) = Span length (ft.)
Beam width = 50-60% of Beam depth.

2. The value of factor β_1:

Concrete strength, f_c'	β_1 value
For f_c' up to 4000 psi	$\beta_1 = 0.85$
For 4000 psi $< f_c' \leq$ 8000 psi	$\beta_1 = 0.85 - 0.05\frac{f_c' - 4000}{1000}$
For $f_c' >$ 8000 psi	$\beta_1 = 0.65$

3. T – beam introduction:



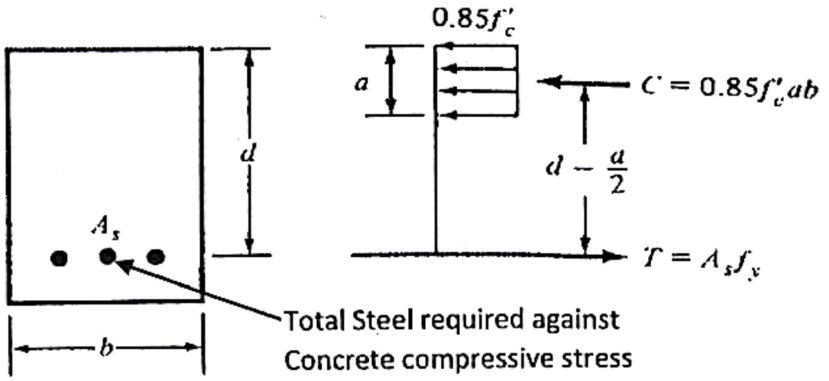
b = Flange width

b_w = Web width

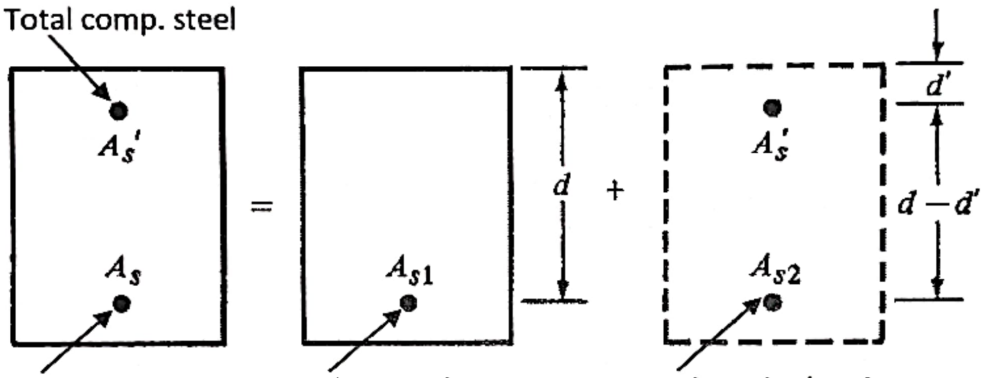
h_f = Height of flange

d = Effective depth

4. Singly Reinforced Beam – Outline:

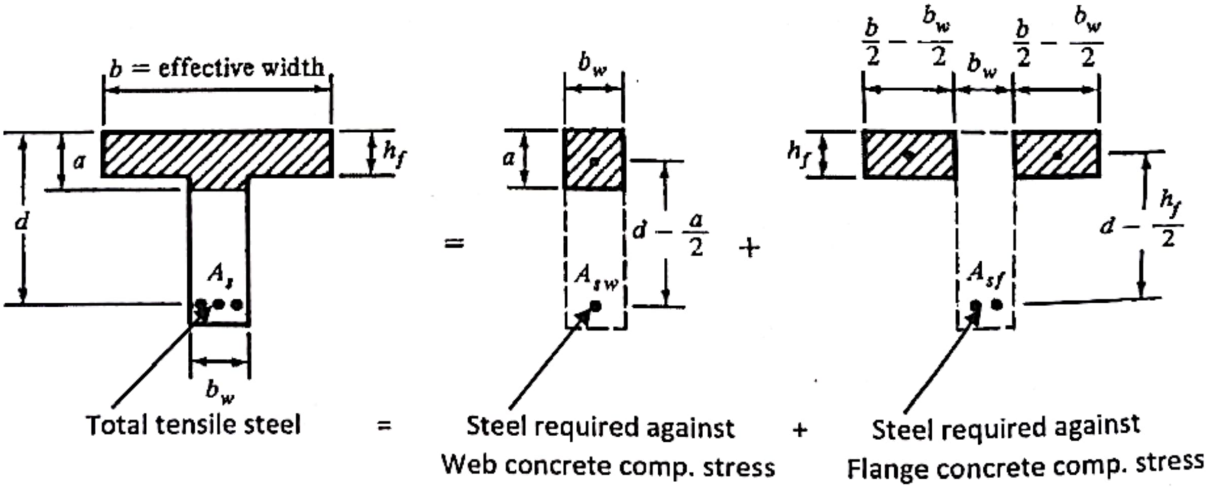


5. Doubly Reinforced Beam – Outline:

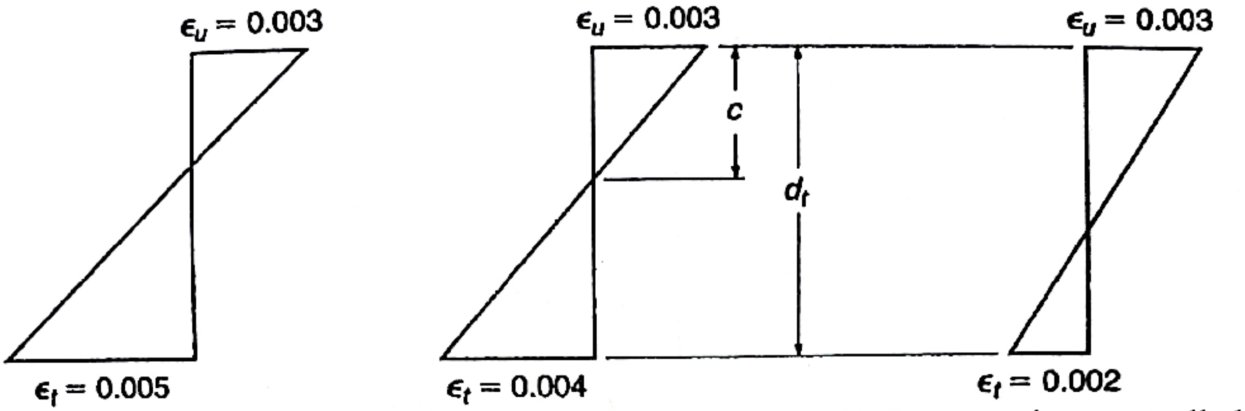


$$M_n = M_{n_1} + M_{n_2} = M_{n_1} = A_{s_1} f_y \left(d - \frac{a}{2}\right) + M_n = A_s' f_s' (d - d') = A_{s_2} f_y (d - d')$$

6. T - Beam – Outline:



7. Tensile strain (ϵ_t) in analysis and design of beams:



(a) Tension-controlled member

(b) Minimum net tensile strain

(c) Compression-controlled member

Figure: Strain diagram of steel and concrete

i) For analysis of beam:

When a beam is to be analysed it is difficult to find the tensile strain of steel (ϵ_t). So, it is considered the minimum net tensile strain, $\epsilon_t = 0.004$.

ii) For design of beam:

When a beam is to be designed it must be provided the fully tension-controlled member so that it can be ensured desired flexural strength. So, it is considered the tension-controlled member strain, $\epsilon_t = 0.005$.

So, it should be always remembered:

Beam Problem	Tensile Strain (ϵ_t)
Analysis	0.004
Design	0.005

2.3. Solution Techniques:

2.3.1 Singly Reinforced Beam - *Analysis*

WSD

Step-1. Design constants:

$$n = \frac{E_s}{E_c}$$

$$r = \frac{f_s}{f_c}$$

$$k = \frac{n}{n+r}$$

$$j = 1 - \frac{k}{3}$$

Step-2. Moment:

If steel controls, $M = A_s f_s j d$

If concrete controls, $M = \frac{1}{2} f_c k j b d^2$

USD

Step-1. Steel ratio:

$$\rho = \frac{A_s}{bd}$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t};\ \epsilon_t = 0.004$$

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y}$$

Step-2.

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

Step-3. Moment:

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$M_n = \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

Step-4. *φ check:*

$$c = \frac{a}{\beta_1};\quad \epsilon_t = \frac{d-c}{c}\epsilon_u$$

$$\therefore \varphi = 0.483 + 83.3\ \epsilon_t$$

Step-5. Design Moment (Max.):

$$M_u = \varphi M_n$$

2.3.2. Singly Reinforced Beam - ***Design***

WSD

Step-1. Trial dimension

Step-2. Load & Moment

Step-3. Design constants:

$$n = \frac{E_s}{E_c}$$

$$r = \frac{f_s}{f_c}$$

$$k = \frac{n}{n+r}$$

$$j = 1 - \frac{k}{3}$$

Step-4. Depth check:

$$M = \frac{1}{2} f_c k j b d^2$$

$$\therefore d_{req.} = \sqrt{\frac{2M_{max}}{f_c k j b}}$$

Here, $b = 10'', 12''$ *&* $so - as\ wish$

Step-5. Steel calculation:

$$A_s = \frac{M}{f_s j d}$$

USD

Step-1. Trial dimension

Step-2. Load & Moment

Step-3. Steel ratio:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t};\ \epsilon_t = 0.005$$

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y}$$

$\rho_{max} > \rho_{min}\ \ (ok)$

ρ_{max} can be used.

Step-4. Depth check:

$$M_u = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

Step-5. Steel calculation:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} \quad \text{----------} \boxed{1}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad \text{----------} \boxed{2}$$

$$\therefore A_s = \surd$$

2.3.3. Doubly RC Beam - *Analysis*

WSD

Step-1. Design constants:

$$n, r, k \,\&\, j$$

Step-2. Check doubly/singly:

$Here,\ A_s = given$

$$M_1 = \frac{1}{2} f_c k j b d^2$$

$$M_1 = A_{s_1} f_s j d$$

$$\therefore A_{s_1} = \surd$$

$A_s > A_{s_1}$; *so doubly* RC beam is required.

Step-3. Moment:

$M_1 = \surd\ (in\ tension\ zone/bottom)$

$$\therefore A_{s_2} = A_s - A_{s_1}$$

$$\therefore f_s{}' = 2 f_s \frac{k - d'/d}{1 - k}$$

$M_2 = A_s{}' f_s{}' (d - d')$; $(in\ comp.zone)$

$M_2 = A_{s_2} f_s (d - d')$; [excess moment in tension zone]

$\therefore M_2 = \longrightarrow smaller$

$\therefore Moment\ capacity, M = M_1 + M_2$

USD

Step-1. Check doubly/singly:

$$\rho = \frac{A_s}{bd}\ ;\ \rho' = \frac{A_s{}'}{bd}$$

$$\rho_{max} = 0.85 \beta_1 \frac{f_c{}'}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t};\ \epsilon_t = 0.004$$

$\because \rho > \rho_{max}$; so doubly beam is required.

Step-2. Check comp. steel yield/not:

$$\bar{\rho}_{cy} = 0.85 \beta_1 \frac{f_c{}'}{f_y} \cdot \frac{d'}{d} \cdot \frac{\epsilon_c}{\epsilon_c - \epsilon_y} + \rho';\ \epsilon_y = \frac{f_y}{E_s}$$

$\because \rho > \bar{\rho}_{cy}$; $comp.steel\ yield$

$$\therefore f_s{}' = f_y$$

Step-3. Moment:

$$a = \frac{(A_s - A_s{}') f_y}{0.85 f_c{}' b}$$

$$M_n = A_s{}' f_y (d - d') + (A_s - A_s{}') f_y \left(d - \frac{a}{2}\right)$$

Step-4. *φ check:*

$$c = \frac{a}{\beta_1}\ ;\quad \epsilon_t = \frac{d - c}{c} \epsilon_u$$

$$\therefore \varphi = 0.483 + 83.3\ \epsilon_t$$

Step-5. Design moment:

$$M_u = \varphi M_n$$

2.3.4 Doubly Reinforced Beam - *Design*

WSD

Step-1. Load

Step-2. Moment, M

Step-3. Design constants:

$$n, r, k \,\&\, j$$

Step-4. Check doubly/singly:

$$d = h - 4''$$

$$M_1 = \frac{1}{2} f_c k j b d^2$$

$\because M > M_1$; *so doubly* RC beam is required.

Step-5. Extra moment:

$$M_2 = M - M_1$$

Step-6. Steel calculation:

Tensile zone (bottom):

$$A_{s_1} = \frac{M_1}{f_s j d}$$

$$M_2 = A_{s_2} f_s (d - d')$$

$$\therefore A_{s_2} = \frac{M_2}{f_s j (d - d')}$$

$$\therefore Total\ steel, A_s = A_{s_1} + A_{s_2}$$

Compression zone (top):

$$\therefore f_s' = 2 f_s \frac{k - d'/d}{1 - k}$$

$$\because f_s' > f_s\ ; use\ f_s.$$

$$A_s' = \frac{M_2}{f_s (d - d')}$$

USD

Step-1. Load: $1.2DL + 1.6LL$

Step-2. Moment, M

Step-3. Check doubly/singly:

$$d = h - 4''$$

$$\rho_{max} = 0.85 \beta_1 \frac{f_c'}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t};\ \epsilon_t = 0.005$$

$$\therefore A_{s_1} = \rho_{max} b d$$

$$a = \frac{A_{s_1} f_y}{0.85 f_c' b}$$

$$M_{n_1} = A_{s_1} f_y \left(d - \frac{a}{2}\right)$$

$$\therefore M_u = \varphi M_{n_1}$$

$\because M > M_u$; so doubly RC beam required.

Step-4. Extra moment:

$$M_{n_2} = \frac{M}{\varphi} - M_{n_1}$$

Step-5. Steel calculation:

$d' = 2.5''$

Compression steel:

$$c = \frac{a}{\beta_1};\ \ \epsilon_s' = \frac{c - d'}{c} \epsilon_c;\ f_s' = \epsilon_s' E_s$$

$$M_{n_2} = A_s' f_s' (d - d')$$

$$\therefore A_s' = \frac{M_{n_2}}{f_s' (d - d')}$$

Tensile steel:

$$A_s = A_{s_1} + A_s' \frac{f_s'}{f_y}$$

Step-6. Check for moment:

2.3.5 T - Beam - *Analysis*

WSD

Step-1. Flange width:

$$b = \frac{L}{4}$$

$$b = 16t + b_w$$

$$b = c/c\ beam\ spacing$$

$$\therefore b = minimum$$

Step-2. Design constants:

$$n = \frac{E_s}{E_c}$$

$$\rho = \frac{A_s}{bd}$$

$$k = \frac{n\rho + \frac{1}{2}\left(\frac{t}{d}\right)^2}{n\rho + \frac{t}{d}}$$

$$j = \frac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^3\left(\frac{1}{2\rho n}\right)}{6 - 3\left(\frac{t}{d}\right)}$$

Step-3. Check T-beam/not:

$$\because kd > t\ ; T - beam\ is\ needed$$

Step-4. Moment:

$$M = A_s f_s jd$$

$$M = f_c\left(1 - \frac{t}{2kd}\right)btjd$$

Step-5. Check f_c:

$$f_c = \frac{M}{\left(1 - \frac{t}{2kd}\right)btjd}$$

$$Flange\ thickness = t = h_f$$

USD

Step-1. Flange width:

$$b = \frac{L}{4}$$

$$b = 16h_f + b_w$$

$$b = c/c\ beam\ spacing$$

$$\therefore b = minimum$$

Step-2. Check T-beam/not:

$$Here, \quad A_s = given$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$\because a > h_f$; T-beam is required.

Step-3. Moment:

$$A_{sf} = \frac{0.85 f_c'(b - b_w)h_f}{f_y}$$

$$\therefore M_{n_1}(flange) = A_{sf} f_y\left(d - \frac{h_f}{2}\right)$$

$$a = \frac{(A_s - A_{sf})f_y}{0.85 f_c' b_w}$$

$$\therefore M_{n_2}(web) = (A_s - A_{sf})f_y\left(d - \frac{a}{2}\right)$$

$$\therefore M_n = M_{n_1} + M_{n_2}$$

Step-4. *φ check:*

$$c = \frac{a}{\beta_1}\ ; \quad \epsilon_t = \frac{d - c}{c}\epsilon_u$$

$$\therefore \varphi = 0.483 + 83.3\,\epsilon_t$$

Step-5. Design moment:

$$M_u = \varphi M_n$$

2.3.6 T - Beam - ***Design***

WSD

Step-1. Flange width:

$$b = \frac{L}{4}$$

$$b = 16t + b_w$$

$$b = c/c\ beam\ spacing$$

$$\therefore b = minimum$$

Step-2. Moment

Step-3. Trial steel area:

$$M = A_s f_s \left(d - \frac{t}{2}\right)$$

$$\therefore A_s = \frac{M}{f_s \left(d - \frac{t}{2}\right)}$$

Step-4. Design constants:

$$n = \frac{E_s}{E_c}$$

$$\rho = \frac{A_s}{bd}$$

$$k = \frac{n\rho + \frac{1}{2}\left(\frac{t}{d}\right)^2}{n\rho + \frac{t}{d}}$$

$$j = \frac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^3 \left(\frac{1}{2\rho n}\right)}{6 - 3\left(\frac{t}{d}\right)}$$

Step-5. Check T-beam/not:

$$\because kd > t\ ;\ T - beam\ is\ needed$$

Step-6. Steel calculation:

$$A_s = \frac{M}{f_s j d}$$

Step-7. Check f_c:

$$f_c = \frac{M}{\left(1 - \frac{t}{2kd}\right) btjd}$$

USD

Step-1. Flange width:

$$b = \frac{L}{4}$$

$$b = 16h_f + b_w$$

$$b = c/c\ beam\ spacing$$

$$\therefore b = minimum$$

Step-2. Moment

Step-3. Trial steel area:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{h_f}{2}\right)}$$

Step-4. Check T-beam/not:

$$\therefore a = \frac{A_s f_y}{0.85 f_c' b}$$

$\because a > h_f$; T-beam is required.

Step-5. Individual Moment:

$$M_u = given\ \surd$$

$$A_{sf} = \frac{0.85 f_c'(b - b_w) h_f}{f_y}$$

$$\therefore M_{n_1}(flange) = A_{sf} f_y \left(d - \frac{h_f}{2}\right)$$

$$M_{u_1} = \varphi M_{n_1}$$

$$\therefore M_{u_2} = M_u - M_{u_1}$$

Step-6. Steel calculation:

$$A_{s_2} = (A_s - A_{sf}) = \frac{M_{u_2}}{\varphi f_y \left(d - \frac{a}{2}\right)} \text{---[1]}$$

$$\&\quad a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b} \text{-------[2]}$$

$$\therefore A_{s_2} = (A_s - A_{sf}) = \surd$$

$$\therefore A_s = A_{sf} + A_{s_2} = \surd$$

7. φ *check:*

$$c = \frac{a}{\beta_1}\ ;\quad \epsilon_t = \frac{d - c}{c}\epsilon_u$$

$$\therefore \varphi = 0.483 + 83.3\,\epsilon_t$$

2.4. Problem Solution:

2.4.1 Singly Reinforced Beam: Analysis - WSD

Pb-1. A rectangular beam has a width of 12 in. and an effective depth to the centroid of the reinforcement bar is 17.5 in. It is reinforced with three No. 8 bars in one row. If material strengths are $f_c' = 4000\ psi$ *and* $f_y = 60{,}000\ psi$, what is the maximum working moment that can be resisted? [*CE-2019(sim), 2018; BECM-2019(sim)*]

Sol^n:

1. Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000 \times \sqrt{4000}} = 8.04 \approx 8$$

$$r = \frac{f_s}{f_c} = \frac{24}{1.8} = 13.33\,; \qquad [f_s = 0.4f_y = 24\ ksi, f_c = 0.45f_c' = 1.8\ ksi]$$

$$k = \frac{n}{n+r} = \frac{8}{8+13.33} = 0.375$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.375}{3} = 0.875$$

2. Moment:

If steel controls, $M = A_s f_s jd = 3 \times 0.79 \times 24 \times 0.875 \times 17.5 = 871\ k - in.$

If concrete controls,

$$M = \frac{1}{2} f_c kjbd^2 = \frac{1}{2} \times 1.8 \times 0.375 \times 0.875 \times 12 \times (17.5)^2 = 1085\ k - in.$$

$\therefore M_{max} = 871\ k - in.$

(Answer)

2.4.2 Singly Reinforced Beam: Analysis - *USD*

Pb-2. A rectangular beam has width of 12 in. and effective depth 17.5 in. It is reinforced with four No. 9 bars in one row. If $f_c' = 4000\ psi$ *and* $f_y = 60{,}000\ psi$, what is the nominal flexural strength and what is the maximum moment that can be utilized in design?

Sol^n:

1. Steel ratio:

$$\rho = \frac{A_s}{bd} = \frac{4 \times 1.0}{12 \times 17.5} = 0.019$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{4}{60} \cdot \frac{0.003}{0.003 + 0.004} = 0.0206$$

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} = \frac{3 \times \sqrt{4000}}{60000} \geq \frac{200}{60000} = 0.0032 \geq 0.0033 \approx 0.0033$$

$\because \rho_{max}$ *can be used*

2. $a = \dfrac{A_s f_y}{0.85 f_c' b} = \dfrac{4 \times 1.0 \times 60}{0.85 \times 3 \times 12} = 5.88''$

3. Moment:

If steel controls,

$$\therefore M_n = A_s f_y \left(d - \frac{a}{2}\right) = 4 \times 1.0 \times 60 \times \left(17.5 - \frac{5.88}{2}\right) = 3494\ k - in.$$

If concrete controls, $M_n = \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$

$$\therefore M_n = 0.0206 \times 60 \times 12 \times (17.5)^2 \times \left(1 - 0.59 \times 0.0206 \times \frac{60}{4}\right) = 3714\ k - in.$$

$\therefore$ Nominal moment, $M_n = 3494\ k - in.$

4. *φ check:*

$$c = \frac{a}{\beta_1} = \frac{5.88}{0.85} = 6.92''$$

$$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{17.5 - 6.92}{6.92} \times 0.003 = 0.0046$$

$$\therefore \varphi = 0.483 + 83.3\ \epsilon_t = 0.483 + 83.3 \times 0.0046 = 0.87$$

5. Design Moment:

$\therefore$ Maximum moment, $M_u = \varphi M_n = 0.87 \times 3494 = 3040\ k - in.$ (***Ans.***)

2.4.3 Singly Reinforced Beam: Design - WSD

Pb-3. A simply supported beam with a span of 18 ft supports a uniform live load of 975 lb/ft and the dead load of a wall weighing 380 lb/ft in addition to its own weight with $f_c' = 3000\ psi$ *and* $f_s = 20{,}000\ psi$. Determine the required steel area. [***BECM-2018(sim)***]

Sol^n:

1. Trial dimension:

Assume, Total depth = 18″ [Let, Beam depth (in.) = Span length (ft)]
Beam width = 10″

2. Load & Moment:

i) Self-weight $= \frac{18\times10}{144} \times 150 = 187.5\ plf$
ii) Live load $= 975\ plf$
iii) Dead load $= 30\ plf$
Total load, $w = 1542.5\ plf$

∴ Moment,

$$M = \frac{wl^2}{8} = \frac{1542.5 \times (18)^2}{8} = 62.47\ k-ft = 749.64\ k-in.$$

3. Design constants:

$$n = \frac{E_s}{E_c} = \frac{29\times10^6}{57000\sqrt{3000}} = 9.29 \approx 9$$

$$r = \frac{f_s}{f_c} = \frac{20000}{0.45\times3000} = 14.81$$

$$k = \frac{n}{n+r} = 0.378$$

$$j = 1 - \frac{k}{3} = 0.874$$

4. Depth check:

$$M = \frac{1}{2} f_c k j b d^2$$

$$\therefore d_{req.} = \sqrt{\frac{2M_{max}}{f_c k j b}} = \sqrt{\frac{2\times749.64}{0.45\times3\times0.378\times0.874\times10}} = 18.33'' \approx 19''$$

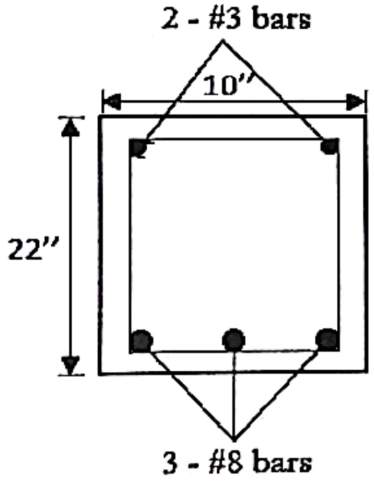
Total depth, $h = 19 + 2.5 = 21.5'' \approx 22''$

∴ Effective depth, $d = 22 - 2.5 = 19.5''$

5. Steel calculation:

$$A_s = \frac{M}{f_s jd} = \frac{749.64}{20 \times 0.874 \times 19.5} = 2.20\ in^2$$

Provide 3 - #8 bars with $A_s = 2.37\ in^2$



2.4.4 Singly Reinforced Beam: Design - *USD*

Pb-4. A singly reinforced beam rectangular beam is to be designed to carry a service live load of 1300 lb/ft. in addition to its own weight, on a 26 ft. simple span. With $f_c' = 3000\ psi$ *and* $f_s = 60{,}000\ psi$, determine the concrete dimensions and steel reinforcement bars. [*CE-2018*]

Sol^n:

1. Trial dimension:

Assume, Total depth $= 26''$ [Let, Beam depth (in.) = Span length (ft)]

Beam width $= 12''$

2. Load & Moment:

i) Self-weight $= \frac{26 \times 12}{144} \times 150 \times 1.2 = 390\ plf$

ii) Live load $= 1300 \times 1.6 = 2080\ plf$

Total load, $w = 2470\ plf = 2.47\ k/ft$

∴ Moment,

$$M = \frac{wl^2}{8} = \frac{2.47 \times (26)^2}{8} = 208.715\ k-ft = 2505\ k-in.$$

3. Steel ratio:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{4}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0181$$

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} = \frac{3 \times \sqrt{4000}}{60000} \geq \frac{200}{60000} = 0.0032 \geq 0.0033 \approx 0.0033$$

$\rho_{max} > \rho_{min}$ (ok)

ρ_{max} can be used.

4. Depth check:

$$M_u = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

$$\Rightarrow 2505 = 0.90 \times 0.0181 \times 60 \times 12 \times d^2 \left(1 - 0.59 \times 0.0181 \times \frac{60}{4}\right)$$

$$\therefore d_{req.} = 15.95 \approx 16''$$

$$\therefore d_{provided} = 26 - 2.5 = 23.5\ in. > d_{req.} \qquad (ok)$$

5. Steel calculation:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$$

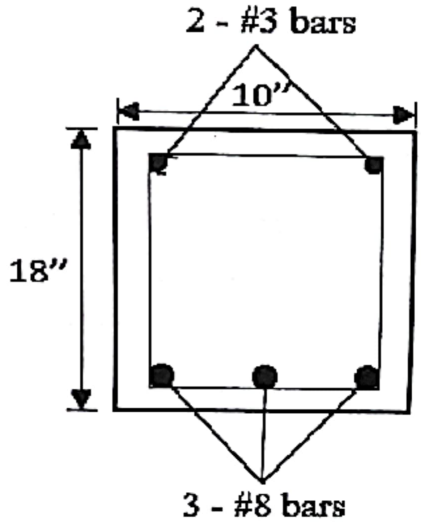
$$\therefore A_s = \frac{2505}{0.90 \times 60 \times \left(23.5 - \frac{a}{2}\right)} \text{ ------ } \boxed{1}$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$\therefore a = \frac{A_s \times 60}{0.85 \times 4 \times 12} \text{ ---------- } \boxed{2}$$

$$\therefore A_s = 2.11\ in^2$$

Provide 3 - #8 bars with $A_s = 2.37\ in^2$



2.4.5 Doubly Reinforced Beam: Analysis - $\boxed{WSD}$

Pb-5. A rectangular beam has a width of 12 in. and an effective depth to the centroid of the tensile reinforcement of 18 in. The tensile reinforcement consists of four No. 10 bars. Compressive reinforcement is provided by three No. 8 bars, the centroid of which is 2.5 in. from the compression face of the beam. If 60,000 psi yield steel is used at a working stress $f_s = 24{,}000\ psi$ *and* 4000 psi concrete is used at an allowable stress of $f_c = 1800\ psi$, what is the allowable working moment that the beam can resist?

Sol^n:

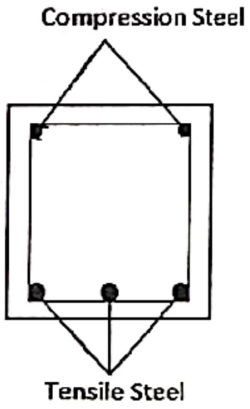
1. Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000 \times \sqrt{4000}} = 8.04 \approx 8$$

$$r = \frac{f_s}{f_c} = \frac{24}{1.8} = 13.33$$

$$k = \frac{n}{n+r} = \frac{8}{8+13.33} = 0.375$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.375}{3} = 0.875$$



2. Check doubly/singly:

Here, $A_s = 4 * 1.27 = 5.08\ in^2$

$$\therefore M_1 = \frac{1}{2} f_c k j b d^2 = \frac{1}{2} \times 1.8 \times 0.375 \times 0.875 \times 12 \times 18^2 = 1148.18\ k - in.$$

Again, $M_1 = A_{s_1} f_s j d$

$$\therefore A_{s_1} = \frac{M_1}{f_s j d} = \frac{1148.18}{24 \times 0.875 \times 18} = 3.04\ in^2$$

$A_s > A_{s_1}$; *so doubly* RC beam is required.

3. Moment:

$M_1 = 1148.18\ k - in.$ $(in\ tension\ zone/bottom)$

$$\therefore A_{s_2} = A_s - A_{s_1} = 5.08 - 3.04 = 2.04\ in^2$$

$$\therefore f_s' = 2 f_s \frac{k - d'/d}{1-k} = 2 \times 24 \times \frac{0.375 - \frac{2.5}{18}}{1 - 0.375} = 18.13\ psi$$

$$M_2 = A_s' f_s' (d - d')$$

$$\Rightarrow M_2 = (3 * 0.79) \times 18.13 \times (18 - 2.5)$$

$$\therefore M_2 = 666.12\ k - in.\ \ (in\ comp.zone/top)$$

Again, $M_2 = A_{s_2} f_s (d - d')$

$\therefore M_2 = 2.04 \times 24 \times (18 - 2.5) = 758.88\ \text{k} - \text{in.}$ [excess moment *in tension zone*]
$\because M_2 = 666.12\ k - in.$

$\therefore Moment\ capacity, M = M_1 + M_2 1148.18 + 666.12 = 1814\ k - in.$

(***Ans.***)

2.4.6 Doubly Reinforced Beam: Analysis - *USD*

Pb-6. A rectangular beam has a width of 12 in. and an effective depth to the centroid of the tensile reinforcement of 24 in. The tensile reinforcement consists of six No. 10 bars in two rows. Compression reinforcement consisting of two No. 8 bars is placed 2.5 in. from the compression face of the beam. If $f_y = 60{,}000\ psi\ and\ f_c' = 5000\ psi$, what is the design moment capacity of the beam?

Sol^n:

1. Check doubly/singly:

$$Tensile\ steel\ ratio, \quad \rho = \frac{A_s}{bd} = \frac{6 * 1.27}{12 \times 24} = 0.0265$$

$$Compression\ steel\ ratio, \quad \rho' = \frac{A_s'}{bd} = \frac{2 * 0.79}{12 \times 24} = 0.0055$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.80 \times \frac{5}{60} \times \frac{0.003}{0.003 + 0.004} = 0.0243$$

$\because \rho > \rho_{max}$; so doubly beam is required.

2. Check comp. steel yield/not:

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{d'}{d} \cdot \frac{\epsilon_c}{\epsilon_c - \epsilon_y} + \rho'$$

$$\Rightarrow \bar{\rho}_{cy} = 0.85 \times 0.80 \times \frac{5}{60} \times \frac{2.5}{24} \times \frac{0.003}{0.003 - 0.00207}; \qquad \left[\epsilon_y = \frac{f_y}{E_s} = \frac{60000}{29 \times 10^6} = 0.00207\right]$$

$\therefore \bar{\rho}_{cy} = 0.0245$

$\because \rho > \bar{\rho}_{cy}$; *comp. steel yield*

$\therefore f_s' = f_y$

$\beta_1 = 0.80$ [See Basic Concepts-2.2(2)]

3. Moment:

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b} = \frac{(6 * 1.27 - 2 * 0.79) \times 60}{0.85 \times 5 \times 12} = 7.11''$$

$$M_n = A_s' f_y (d - d') + (A_s - A_s') f_y \left(d - \frac{a}{2}\right)$$

$$\Rightarrow M_n = 1.58 \times 60 \times (24 - 2.5) + (7.62 - 1.58) \times 60 \times \left(24 - \frac{7.11}{2}\right)$$

$$\therefore M_n = 9450\ k - in.$$

4. *φ check:*

$$c = \frac{a}{\beta_1} = \frac{7.11}{0.80} = 8.89''$$

$$\epsilon_t = \frac{d - c}{c}\,\epsilon_u = \frac{24 - 8.89}{8.89} \times 0.003 = 0.0051$$

$$\therefore \varphi = 0.90$$

5. Design moment:

$$M_u = \varphi M_n = 0.90 \times 9450 = 8505\ k - in. \quad (\textbf{\textit{Ans.}})$$

2.4.7 Doubly Reinforced Beam: Design - *WSD*

Pb-7. A rectangular beam is limited by architectural considerations to a width of 13 in. and a total depth of 24 in. It must resist a total working moment of 2530 k-in. If $f_s = 20{,}000\ psi$ & $f_c' = 4000\ psi$, what reinforcement is required for flexural design?

[*BECM-2017*]

Sol^n:

1. Load: *[Direct moment is given]*

2. Moment, M = 2530 k-in.

3. Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000 \times \sqrt{4000}} = 8.04 \approx 8$$

$$r = \frac{f_s}{f_c} = \frac{20}{1.8} = 11.11$$

$$k = \frac{n}{n + r} = \frac{8}{8 + 11.11} = 0.418$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.418}{3} = 0.861$$

4. Check doubly/singly:

$$d = h - 4'' = 24 - 4 = 20''$$

$$\therefore M_1 = \frac{1}{2} f_c k j b d^2 = \frac{1}{2} \times 1.8 \times 0.418 \times 0.861 \times 13 \times (20)^2 = 1684\ k - in.$$

$\because M > M_1$; *so doubly* RC beam is required.

5. Extra moment:

$M_2 = M - M_1 = 2530 - 1684 = 846\ k - in.$

6. Steel calculation:

Tensile zone (bottom):

$$\therefore A_{s_1} = \frac{M_1}{f_s jd} = \frac{1684}{20 \times 0.861 \times 20} = 4.89\ in^2$$

$$M_2 = A_{s_2} f_s (d - d')$$

$$\therefore A_{s_2} = \frac{M_2}{f_s (d - d')} = \frac{846}{20 \times (20 - 2.5)} = 2.42\ in^2$$

$$\therefore Total\ steel, A_s = A_{s_1} + A_{s_2} = 4.89 + 2.42 = 7.31\ in^2$$

Compression zone (top):

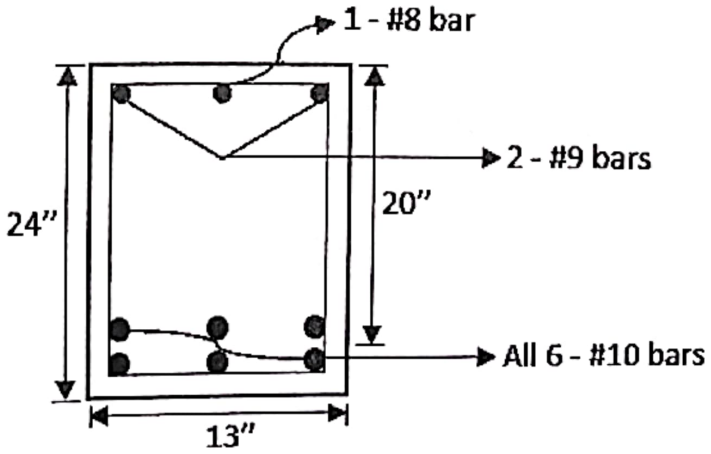
$$\therefore f_s' = 2 f_s \frac{k - d'/d}{1 - k} = 2 \times 20 \times \frac{0.418 - \frac{2.5}{20}}{1 - 0.418} = 20.14\ psi$$

$\because f_s' > f_s$; *use* f_s.

$$A_s' = \frac{M_2}{f_s (d - d')} = \frac{846}{20 \times (20 - 2.5)} = 2.42\ in^2$$

Use 6 - #10 bars in two rows in tensile zone.

Use 1 - #8 & 2 - #9 bars in one row in compression zone.



2.4.8 Doubly Reinforced Beam: Design - *USD*

Pb-8. A rectangular beam that must carry a service live load of 2.47 k/ft and a calculated dead load of 1.05 k/ft on an 18 ft simple span is limited in cross section for architectural reasons to 10 in. width and 20 in. total depth. If $f_y = 60{,}000\ psi$ *and* $f_c' = 4000\ psi$, what steel area must be provided? [***BECM-2019(sim), 2018(sim), 2017(sim)***]

Sol^n:

1. Load, $w = 1.2DL + 1.6LL = 1.2 * 1.05 + 1.6 * 2.47 = 5.21\ k/ft$

2. Moment, $M = \frac{wl^2}{8} = \frac{5.21 \times 18^2}{8} = 211\ k-ft = 2532\ k-in.$

3. Check doubly/singly:

$d = h - 4'' = 20 - 4 = 16''$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{4}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0181$$

$$\therefore A_{s_1} = \rho_{max} bd = 0.0181 \times 10 \times 16 = 2.90\ in^2$$

$$a = \frac{A_{s_1} f_y}{0.85 f_c' b} = \frac{2.90 \times 60}{0.85 \times 4 \times 10} = 5.12''$$

$$M_{n_1} = A_{s_1} f_y \left(d - \frac{a}{2}\right) = 2.90 \times 60 \times \left(16 - \frac{5.12}{2}\right) = 2338\ k-in.$$

$\therefore M_u = \varphi M_{n_1} = 0.90 \times 2338 = 2104\ k-in.$

$\because M > M_u$; *so doubly* RC beam is required.

4. Extra moment:

$$M_{n_2} = \frac{M}{\varphi} - M_{n_1} = \frac{2532}{0.90} - 2338 = 475\ k-in.$$

5. Steel calculation:

$d' = 2.5'', \quad d = 16''$

Compression steel:

$$c = \frac{a}{\beta_1} = \frac{5.12}{0.85} = 6.02''$$

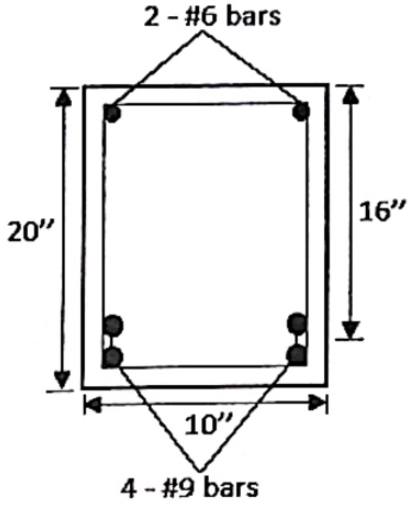
$$\epsilon_s' = \frac{c - d'}{c} \epsilon_c = \frac{6.02 - 2.5}{6.02} \times 0.003 = 0.00175$$

$$f_s' = \epsilon_s' E_s = 0.00175 \times 29 \times 10^6 = 50\ ksi$$

$$M_{n_2} = A_s' f_s' (d - d')$$

$$\therefore A_s{}' = \frac{M_{n2}}{f_s{}'(d-d')} = \frac{475}{50 \times (16-2.5)} = 0.704\ in^2$$

Use 2 - #6 bars with $A_s = 0.88\ in^2$



Tensile steel:

$$A_s = A_{s_1} + A_s{}' \frac{f_s{}'}{f_y} = 2.90 + 0.704 \times \frac{50}{60} = 3.49\ in^2$$

Use 4 - #9 bars with $A_s = 4.0\ in^2$

So, use 2 - #6 bars in one row in compression zone
And, use 4 - #9 bars in two rows in tensile zone.

6. Check for moment:

$$A_{s_1} = A_s - A_s{}' \frac{f_s{}'}{f_y} = 4.0 - 0.88 \times \frac{50}{60} = 3.27\ in^2 > 2.90\ in^2$$

$$a = \frac{A_{s_1} f_y}{0.85 f_c{}' b} = \frac{3.27 \times 60}{0.85 \times 4 \times 10} = 5.77''$$

$$c = \frac{a}{\beta_1} = \frac{5.77}{0.85} = 6.79''$$

$$\epsilon_s{}' = \frac{c-d'}{c}\epsilon_c = \frac{6.79-2.5}{6.79} \times 0.003 = 0.0019$$

$$f_s{}' = \epsilon_s{}' E_s = 0.0019 \times 29 \times 10^6 = 55\ ksi > 50\ ksi$$

So φ will be changed.

$$\therefore d_t = 16 + 1.25 = 17.25''$$

$$\epsilon_t = \frac{d_t - c}{c}\epsilon_u = \frac{17.25-6.79}{6.79} \times 0.003 = 0.0046$$

$$\therefore \varphi = 0.483 + 83.3\,\epsilon_t = 0.483 + 83.3 \times 0.0046 = 0.87$$

$$\textit{Now,} \qquad M_n = A_{s_1} f_y \left(d - \frac{a}{2}\right) + A_s{}' f_s{}' (d-d')$$

$$\Rightarrow M_n = 3.27 \times 60 \times \left(16 - \frac{5.77}{2}\right) + 0.88 \times 55 \times (16-2.5)$$

$$\therefore M_n = 3227\ k-in.$$

$$\because M_u = \varphi M_n = 0.87 \times 3227 = 2807\ k-in > 2532\ k-in. \qquad (ok)$$

2.4.9 T - Beam: Analysis - $\boxed{WSD}$

Pb-9. A floor slab 4 in. thick is supported by reinforced concrete beams 9 ft c/c which, together with the slab, act as T-beams. The beams are simply supported and their span is 18 ft. The cross-section of each beam below the slab is 10 in. × 20 in.; the reinforcement consists of six No. 8 bars in two rows, 2 in. c/c vertically, the center of the lower row being 2.5 in. above the lower surface of the beam. If $f_s = 20{,}000\ psi$ & $f_c' = 2500\ psi$, what is the maximum allowable working moment of the beam?

Sol^n:

1. Flange width:

$$b = \frac{L}{4} = \frac{18 \times 12}{4} = 54''$$

$$b = 16t + b_w = 16 \times 4 + 10 = 74''$$
$$b = c/c\ beam\ spacing = 9 \times 12 = 108''$$

$$\therefore b = 54''$$

2. Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{2500}} = 10$$

$$\rho = \frac{A_s}{bd} = \frac{6 * 0.79}{54 \times 20.5} = 0.00428 \quad | \quad d = 20 + 4 - 2.5 - \frac{2}{2} = 20.5''$$

$$k = \frac{n\rho + \frac{1}{2}\left(\frac{t}{d}\right)^2}{n\rho + \left(\frac{t}{d}\right)} = \frac{10 \times 0.00428 + \frac{1}{2}\left(\frac{4}{20.5}\right)^2}{10 \times 0.00428 + \left(\frac{4}{20.5}\right)} = 0.256$$

$$j = \frac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^3\left(\frac{1}{2\rho n}\right)}{6 - 3\left(\frac{t}{d}\right)} = \frac{6 - 6\left(\frac{4}{20.5}\right) + 2\left(\frac{4}{20.5}\right)^2 + \left(\frac{4}{20.5}\right)^3\left(\frac{1}{2 \times 0.00428 \times 10}\right)}{6 - 3\left(\frac{4}{20.5}\right)}$$

$$\therefore j = 0.922$$

3. Check T-beam/not:

$$\because kd = 0.256 \times 20.5 = 5.25'' > t(= 4'');\ T - beam\ is\ needed$$

4. Moment:

$$M = A_s f_s j d = (6 * 0.79) \times 20 \times 0.922 \times 20.5 = 1792\ k - in.$$

$\& \; M = f_c\left(1-\frac{t}{2kd}\right)btjd \quad | \quad f_c = 0.45f_c' = 1125\ psi$

$\therefore M = 1.125\left(1-\frac{4}{2\times 0.256\times 20.5}\right)\times 54\times 4\times 0.922\times 20.5 = 2843\ k-in.$

$\therefore M_{allowable} = 1792\ k-in.$

5. Check f_c:

$$f_c = \frac{M}{\left(1-\frac{t}{2kd}\right)btjd} = \frac{1792}{\left(1-\frac{4}{2\times 0.256\times 20.5}\right)\times 54\times 4\times 0.922\times 20.5} = 709\ psi$$

But, available $f_c = 1125\ psi > 709\ psi$ (ok).

2.4.10 T - Beam: Analysis - *USD*

Pb-10. The isolated T-beam is composed of a flange 28 in. wide and 6 in. deep cast monolithically with a web of 10 in. width that extends 24 in. below the bottom surface of the flange to produce a beam of 30 in. total depth. Tensile reinforcement consists of six No. 10 bars placed in two rows at 1 in. vertical spacing. The centroid of the bar group is 26 in. from the top of the beam. If $f_c' = 3000\ psi$ *and* $f_y = 60{,}000\ psi$, what is the design moment capacity of the beam? **[*CE-2016*]**

Sol^n:

1. Flange width:

Given, $b = 28''$

2. Check T-beam/not:

Here, $A_s = 6 * 1.27 = 7.62\ in^2$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{7.62\times 60}{0.85\times 3\times 28} = 6.4'' > h_f (= 6'')$$

$\because a > h_f$; T-beam is required.

3. Moment:

$$A_{sf} = \frac{0.85 f_c'(b-b_w)h_f}{f_y} = \frac{0.85\times 3\times (28-10)\times 6}{60} = 4.59\ in^2$$

$$\therefore M_{n_1}(flange) = A_{sf} f_y\left(d-\frac{h_f}{2}\right) = 4.59\times 60\times\left(26-\frac{6}{2}\right) = 6334\ k-in.$$

$$a = \frac{(A_s - A_{sf})f_y}{0.85f_c{}'b_w} = \frac{(7.62 - 4.59) \times 60}{0.85 \times 3 \times 10} = 7.13''$$

$$\therefore M_{n_2}(web) = (A_s - A_{sf})f_y\left(d - \frac{a}{2}\right) = (7.62 - 4.59) \times 60 \times \left(26 - \frac{7.13}{2}\right) = 4079\ k - in.$$

$$\therefore M_n = M_{n_1} + M_{n_2} = 6334 + 4079 = 10{,}413\ k - in.$$

4. *φ check:*

$$c = \frac{a}{\beta_1} = \frac{7.13}{0.85} = 8.39''$$

$$\in_t = \frac{d - c}{c} \in_u = \frac{26 - 8.39}{8.39} \times 0.003 = 0.0063 > 0.005$$

$$\therefore \varphi = 0.90$$

5. Design moment:

$$M_u = \varphi M_n = 0.90 \times 10{,}413 = 9372\ k - in.$$

2.4.11 T - Beam: Design - *WSD*

Pb-11. A floor system consists of a 3 in. concrete slab supported by continuous T-beams of 24 ft. spans, 47″ on centers. If $b_w = 11''$ *and* $d = 20''$, what tensile steel area is required at midspan to resist a working moment of 2500 k-in. if $f_s = 20{,}000\ psi$ & $f_c{}' = 3000\ psi$?

Sol^n:

1. Flange width:

$$b = \frac{L}{4} = \frac{24 \times 12}{4} = 72''$$

$$b = 16t + b_w = 16 \times 3 + 11 = 59''$$

$$b = c/c\ beam\ spacing = 47''$$

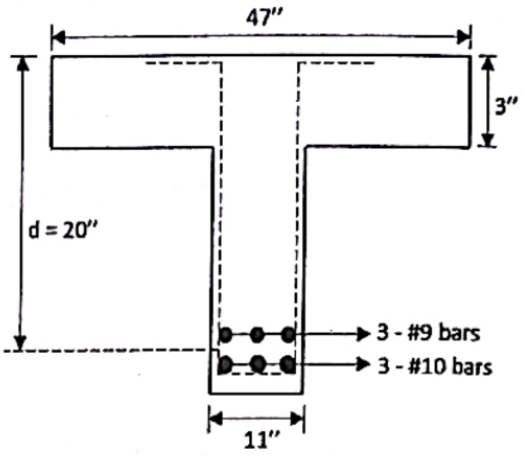
$$\therefore b = 47''$$

2. Moment, $M = 2500\ k-in.$

3. Trial steel area:

$$M = A_s f_s \left(d - \frac{t}{2}\right)$$

$$\therefore A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)} = \frac{2500}{20 \times \left(20 - \frac{3}{2}\right)} = 6.76\ in^2$$



4. Design constants:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9$$

$$\rho = \frac{A_s}{bd} = \frac{6.76}{47 \times 20} = 0.0072$$

$$k = \frac{n\rho + \frac{1}{2}\left(\frac{t}{d}\right)^2}{n\rho + \left(\frac{t}{d}\right)} = \frac{9 \times 0.0072 + \frac{1}{2}\left(\frac{3}{20}\right)^2}{n\rho + \left(\frac{3}{20}\right)} = 0.354$$

$$j = \frac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^3\left(\frac{1}{2\rho n}\right)}{6 - 3\left(\frac{t}{d}\right)} = \frac{6 - 6\left(\frac{3}{20}\right) + 2\left(\frac{3}{20}\right)^2 + \left(\frac{3}{20}\right)^3\left(\frac{1}{2 \times 0.0072 \times 9}\right)}{6 - 3\left(\frac{3}{20}\right)}$$

$$\therefore j = 0.928$$

5. Check T-beam/not:

$\because kd = 0.354 \times 20 = 7.08'' > t(= 3'');\ \ T - beam\ is\ needed$

6. Steel calculation:

$$A_s = \frac{M}{f_s jd} = \frac{2500}{20 \times 0.928 \times 20} = 6.73\ in^2 < A_{s_{trial}}$$

$\therefore A_s = 6.76\ in^2$

Use 3 - #10 bars & 3 - #9 bars in two rows.

7. Check f_c:

$$f_c = \frac{M}{\left(1 - \frac{t}{2kd}\right)btjd} = \frac{2500}{\left(1 - \frac{3}{2 \times 0.354 \times 20}\right) \times 47 \times 3 \times 0.928 \times 20} = 1212\ psi$$

But, available $f_c = 0.45 f_c' = 1350\ psi > 709\ psi$ (ok).

2.4.12 T - Beam: Design - USD

Pb-12. A floor system consists of 3 in. concrete slab and supported by continuous T-beams with a 24 ft. span, 47″ on centers of beams. If $b_w = 11''$ *and* $d = 20''$, what tensile reinforcement is needed at midspan to resist a factored moment of 6400 k-in. if $f_y = 60{,}000\ psi$ & $f_c' = 3000\ psi$? [*CE-2014; BECM-2019(sim), 2018, 2017*]

Sol^n:

1. Flange width:

$$b = \frac{L}{4} = \frac{24 \times 12}{4} = 72''$$
$$b = 16h_f + b_w = 16 \times 3 + 11 = 59''$$
$$b = c/c\ beam\ spacing = 47''$$
$$\therefore b = 47''$$

2. Moment, $M_u = 6400\ k - in.$

3. Trial steel area:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{h_f}{2}\right)} = \frac{6400}{0.90 \times 60 \times \left(20 - \frac{3}{2}\right)} = 6.41\ in^2$$

4. Check T-beam/not:

$$\therefore a = \frac{A_s f_y}{0.85 f_c' b} = \frac{6.41 \times 60}{0.85 \times 3 \times 47} = 3.21'' > h_f$$

$\because a > h_f$; T-beam is required.

5. Individual Moment:

$M_u = 6400\ k - in.$

$$A_{sf} = \frac{0.85 f_c'(b - b_w)h_f}{f_y} = \frac{0.85 \times 3 \times (47 - 11) \times 3}{60} = 4.59\ in^2$$

$$\therefore M_{n_1}(flange) = A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 4.59 \times 60 \times \left(20 - \frac{3}{2}\right) = 5095\ k - in.$$

$M_{u_1} = \varphi M_{n_1} = 0.90 \times 5095 = 4585\ k - in.$

$\therefore M_{u_2} = M_u - M_{u_1} = 6400 - 4585 = 1815\ k - in.$

M_{u_2} will be resisted by rectangular (web) section.

6. Steel calculation:

$$A_{s_2} = (A_s - A_{sf}) = \frac{M_{u_2}}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{1815}{0.90 \times 60 \times \left(20 - \frac{a}{2}\right)} - - - - -[1]$$

$$\& \quad a = \frac{(A_s - A_{sf}) \times 60}{0.85 \times 3 \times 11} - - - - - - -[2]$$

$\therefore A_{s_2} = (A_s - A_{sf}) = 1.88\ in^2$

$\therefore A_s = A_{sf} + A_{s_2} = 4.59 + 1.88 = 6.47\ in^2 > A_{s_{trial}}$

$\therefore A_s = 6.47\ in^2$

Provide 3 - #10 bars & 3 - #9 bars in two rows.

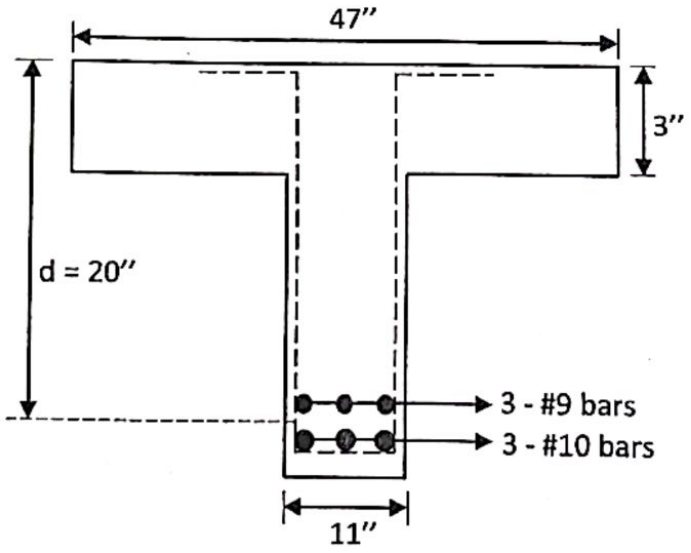
7. *φ check:*

$$a = \frac{(A_s - A_{sf}) \times 60}{0.85 \times 3 \times 11} = \frac{1.88 \times 60}{0.85 \times 3 \times 11} = 4.02''$$

$$c = \frac{a}{\beta_1} = \frac{4.02}{0.85} = 4.73''$$

$$\epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{20 - 4.73}{4.73} \times 0.003 = 0.0097 > 0.005$$

$\therefore \varphi = 0.90 \qquad (ok).$



2.5. Additional Problems:

Pb-13. A reinforced concrete beam was tested to failure and had a rectangular section with b = 14 in. and d = 18.5 in. At ultimate (failure) moment, the strain in the tension steel was recorded as 0.004106. The strain in the concrete at failure may be assumed to be 0.003. If $f_c' = 3\ ksi\ and\ f_y = 60\ ksi$,
(i) Check if the tension steel has yielded.
(ii) Calculate the steel area provided in the section to develop the above strain. Then calculate the applied moment.
(iii) Calculate the design moment strength of the beam section. [*CE-2019, 2018*]
Sol^n:
(i) Yield strain,

$$\epsilon_y = \frac{f_y}{E_s} = \frac{60000}{29 \times 10^6} = 0.00207$$

Measured strain, $\epsilon_u = 0.004106$
$\therefore \epsilon_u > \epsilon_y$; The tension steel has yielded.

(ii) From the triangle of strain diagram we have,

$$\frac{c}{d} = \frac{0.003}{0.003 + 0.004106} = 0.422$$

$$\therefore c = 0.422 \times d = 0.422 \times 18.5 = 7.81''$$

$$Again, \quad a = \beta_1 c = 0.85 \times 7.81 = 6.64''$$

$$\text{Now, } a = \frac{A_s f_y}{0.85 f_c' b}$$

$$\therefore A_s = \frac{0.85 f_c' ab}{f_y} = \frac{0.85 \times 3 \times 6.64 \times 14}{60} = 3.95\ in^2$$

The section has 3 - #8 bars & 3 - #7 bars in two rows.
Applied moment,

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 3.95 \times 60 \times \left(18.5 - \frac{6.64}{2}\right) = 3598\ k - in.$$

(iii)

$$\epsilon_t = \frac{d_t - c}{c} \epsilon_u = \frac{(18.5 + 1) - 7.81}{7.81} \times 0.003 = 0.00449$$

$$\therefore \varphi = 0.483 + 83.3\ \epsilon_t = 0.483 + 83.3 \times 0.00449 = 0.857$$

$$M_u = \varphi M_n = 0.857 \times 3598 = 3083\ k - in. \qquad (\boldsymbol{Ans}).$$

Pb-14. Singly Reinforced Beam (cantilever beam): Design – *USD:* A singly reinforced rectangular beam is to be designed to carry a service live load of 1.5 k/ft. in addition to its own weight on a 6 ft. *cantilever beam.* Assume, $f_c' = 5000\ psi$ *and* $f_y = 60000\ psi$. Determine the concrete dimensions and steel reinforcement bars. Also include a sketch of cross-section allowing for No. 3 stirrups. [***CE-2019***]

Sol^n:

1. Trial dimension:

For cantilever beam according to ACI code,

Assume, Total depth, $h = \frac{L}{8} = \frac{6\times12}{8} = 9''$

Beam width, $b = 9''$

N.B: Minimum Beam Size: 9" × 9"

2. Load & Moment:

i) Self-weight $= \frac{9\times9}{144} \times 150 \times 1.2 = 101.25\ plf$

ii) Live load $= 1500 \times 1.6 = 2400\ plf$

Total load, $w = 2501.25\ plf = 2.5\ k/ft$

∴ Moment, for *cantilever beam,*

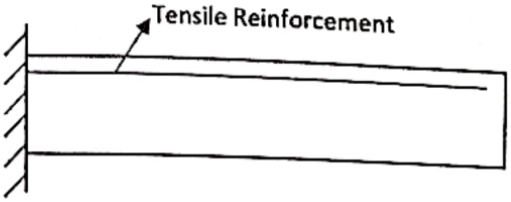
$$M = \frac{wl^2}{2} = \frac{2.5\times(6)^2}{2} = 45\ k-ft = 540\ k-in.$$

3. Steel ratio:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y}.\frac{\epsilon_u}{\epsilon_u+\epsilon_t} = 0.85\times0.80\times\frac{5}{60}\times\frac{0.003}{0.003+0.005} = 0.0213$$

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} = \frac{3\times\sqrt{4000}}{60000} \geq \frac{200}{60000} = 0.0032 \geq 0.0033$$

∴ $\rho_{min} = 0.0033$



$\rho_{max} > \rho_{min}$ (ok)

ρ_{max} can be used.

4. Depth check:

$$M_u = \varphi\rho f_y b d^2\left(1-0.59\rho\frac{f_y}{f_c'}\right)$$

$$\Rightarrow 540 = 0.90\times0.0213\times60\times9\times d^2\left(1-0.59\times0.0213\times\frac{60}{5}\right)$$

∴ $d_{req.} = 7.84 \approx 8''$

∴ $d_{provided} = 9-2.5 = 6.5\ in. < d_{req.}$ $(Not\ ok)$

Revised, Total depth, $h = 8 + 2.5 = 10.5 \approx 11''$
$\therefore$ Effective depth, $d = 11 - 2.5 = 8.5''$

5. Steel calculation:

In cantilever beam, negative moment occurs and tension acts at top. So steel bar is provided in the top of the beam.

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$$

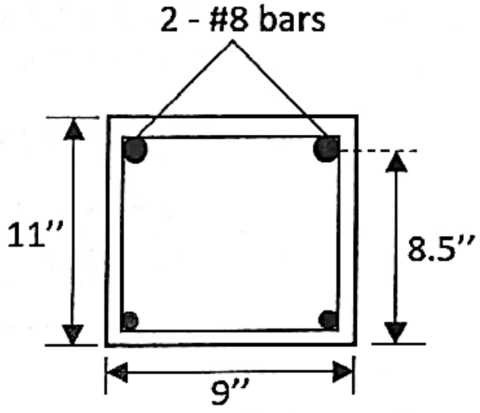
$$\therefore A_s = \frac{540}{0.90 \times 60 \times \left(8.5 - \frac{a}{2}\right)} \text{ ------ } \boxed{1}$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

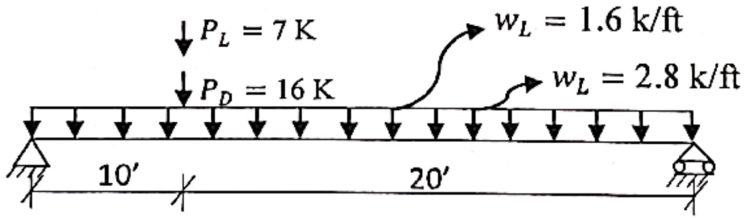
$$\therefore a = \frac{A_s \times 60}{0.85 \times 5 \times 9} \text{ ---------- } \boxed{2}$$

$\therefore A_s = 1.34\ in^2$

Provide 2 - #8 bars with $A_s = 1.58\ in^2$



Pb-15. Singly Reinforced Beam: Design – *USD:* A simply supported rectangular beam carries a uniform service live load of 2.8 k/ft. and a dead load of 1.6 k/ft. in addition to its own weight on an 30 ft. simple span, also the beam supports a concentrated dead load of 16 K and a concentrated live load of 7 K at the point, 10 ft. from the left support. If $f_y = 60\ ksi$ *and* $f_c' = 4\ ksi$, what steel area must be provided? [***BECM-2019(similar)***]



Sol^n:

1. Trial dimension:

Assume, Total depth = 30'' [Let, Beam depth (in.) = Span length (ft)]
Beam width = 20''

2. Load & Moment:

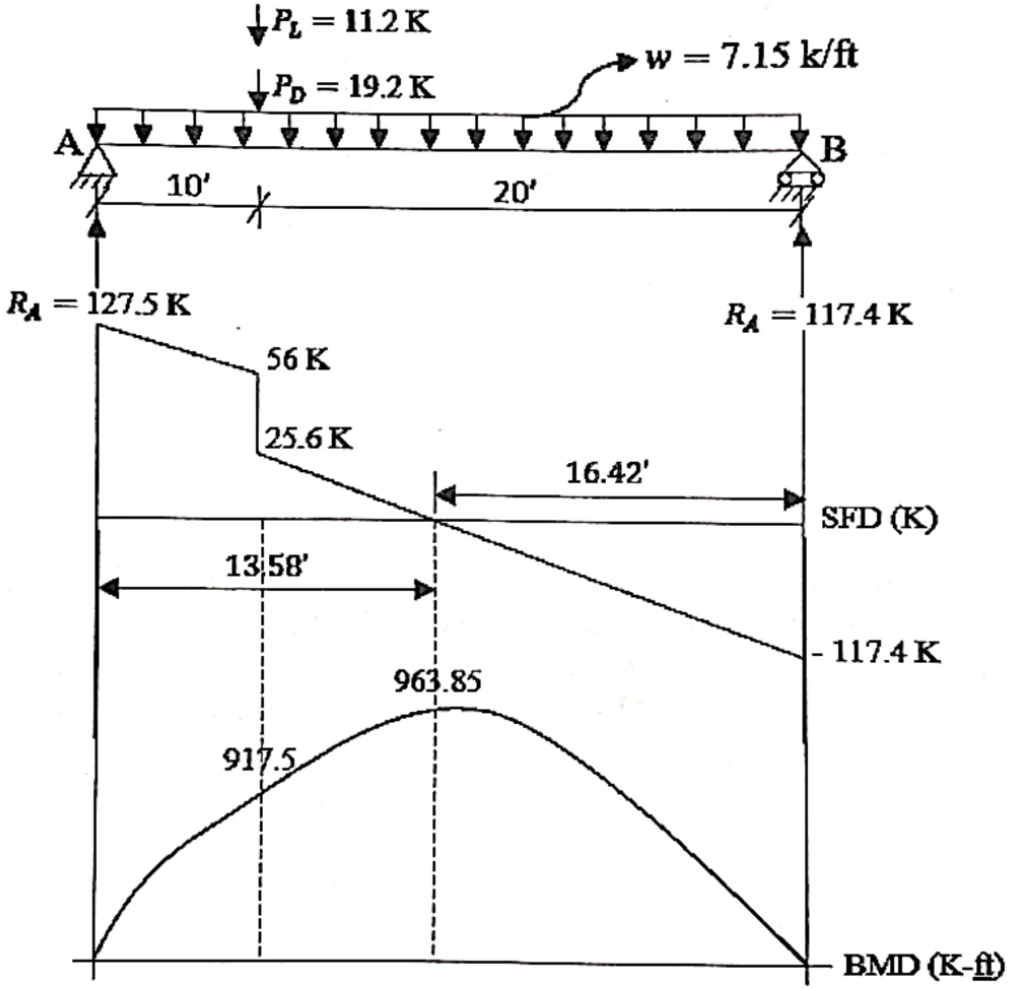
i) Self-weight $= \frac{30 \times 20}{144} \times 150 \times 1.2 = 750\ plf = 0.75\ k/ft$

ii) Dead load $= 1.2 \times 1.6 = 1.92\ k/ft$

iii) Live load $= 1.6 \times 2.8 = 4.48\ k/ft$

Total load, $w = 7.15\ k/ft$

Again, concentrated dead load $= 1.2 \times 16 = 19.2\ K$
& concentrated live load $= 1.6 \times 7 = 11.2\ K$



∴ Maximum moment occurs at zero shear,

$M_{max} = 963.85\ k - ft = 11{,}566\ k - in.$

3. Steel ratio:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{4}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0181$$

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} = \frac{3 \times \sqrt{4000}}{60000} \geq \frac{200}{60000} = 0.0032 \geq 0.0033 \approx 0.0033$$

$\rho_{max} > \rho_{min}$ (ok)

ρ_{max} can be used.

4. Depth check:

$$M_u = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

$$\Rightarrow 11{,}566 = 0.90 \times 0.0181 \times 60 \times 20 \times d^2 \left(1 - 0.59 \times 0.0181 \times \frac{60}{4}\right)$$

$\therefore d_{req.} = 26.54 \approx 27''$

$\therefore d_{provided} = 30 - 2.5 = 27.5\ in. > d_{req.}$ (ok)

5. Steel calculation:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$$

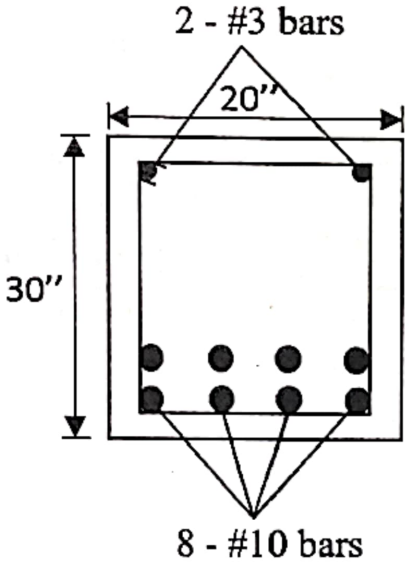
$$\therefore A_s = \frac{11{,}566}{0.90 \times 60 \times \left(27.5 - \frac{a}{2}\right)} \text{---} \boxed{1}$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$\therefore a = \frac{A_s \times 60}{0.85 \times 4 \times 20} \text{---------} \boxed{2}$$

$\therefore A_s = 9.12\ in^2$

Provide 8 - #10 bars in two rows.



Pb-16. T – Beam (False): Analysis – ***USD:*** A series of concrete beams spaced at 9 ft on centers are acting on a simply supported span of 18 ft. The beam supports a reinforced concrete floor slab 4 in. thick. If the width of the web is 10 in. and d = 18 in.; the beam is reinforced with three No. 9 bars. Determine the moment strength of a typical interior beam. Assume $f_c' = 4\ ksi\ and\ f_y = 60\ ksi$. **[*CE-2018*]**

Sol^n:

1. Flange width:

$$b = \frac{L}{4} = \frac{18 \times 12}{4} = 54''$$
$$b = 16t + b_w = 16 \times 4 + 11 = 74''$$
$$b = c/c\ beam\ spacing = 9 \times 12 = 108''$$
$$\therefore b = 54''$$

2. Check T-beam/not:

$$Here, \quad A_s = 3 * 1.0 = 3\ in^2$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3 \times 60}{0.85 \times 4 \times 54} = 0.98'' \not> h_f (= 4'')$$

$\because a \not> h_f$; it is a false T-beam, so singly reinforced beam analysis is needed.

3. Steel ratio:

$$\rho = \frac{A_s}{bd} = \frac{3}{54 \times 18} = 0.0031$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{4}{60} \cdot \frac{0.003}{0.003 + 0.004} = 0.0206$$

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} = \frac{3 \times \sqrt{4000}}{60000} \geq \frac{200}{60000} = 0.0032 \geq 0.0033 \approx 0.0033$$

$\because \rho_{max}\ can\ be\ used$

4. $a = 0.98''$

5. Moment:

If steel controls,

$$\therefore M_n = A_s f_y \left(d - \frac{a}{2}\right) = 3 \times 60 \times \left(18 - \frac{0.98}{2}\right) = 3152\ k - in.$$

6. *φ check:*

$$c = \frac{a}{\beta_1} = \frac{0.98}{0.85} = 1.15''$$

$$\epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{18-1.15}{1.15} \times 0.003 = 0.044 > 0.005$$

$\therefore \varphi = 0.90$

7. Design Moment:

∴ Maximum moment, $M_u = \varphi M_n = 0.90 \times 3152 = 2837\ k-in.$ (***Ans.***)

Pb-17. T – Beam (False): Design – ***USD:*** A concrete floor system consists of parallel T-beams spaced at 10 ft on centers and spanning over 30 ft between supports. The 5 in. thick slab is casted monolithically with T-beam webs having width of 14 in. and total depth of 28 in. In addition to its own weight, each T-beam must carry a superimposed dead load of 50 psf and a service live load of 220 psf. Assume $f_c' = 4\ ksi\ and\ f_y = 60\ ksi$. Determine the required tensile area with neat sketch using USD method. [***CE-2019***]

Sol^n:

1. Flange width:

$$b = \frac{L}{4} = \frac{30 \times 12}{4} = 90''$$

$$b = 16h_f + b_w = 16 \times 5 + 14 = 94''$$

$$b = c/c\ beam\ spacing = 10 \times 12 = 120''$$

$\therefore b = 90''$

2. Load & Moment:

i) Self-weight $= \frac{(90\times5)+(14\times28)}{144} \times 150 \times 1.2 = 1052.5\ plf$

ii) Dead load $= 1.2 \times 50 \times 1' = 60\ plf$

iii) Live load $= 1.6 \times 220 \times 1' = 352\ plf$

Total load, $w = 1464.5\ plf = 1.5\ k/ft$

∴ Moment,

$$M = \frac{wl^2}{8} = \frac{1.5 \times (30)^2}{8} = 168.75\ k-ft = 2025\ k-in.$$

3. Trial steel area:

Here, d = 28 - 2.5 = 25.5″

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{h_f}{2}\right)} = \frac{2025}{0.90 \times 60 \times \left(25.5 - \frac{5}{2}\right)} = 1.63\ in^2$$

4. Check T-beam/not:

$$\therefore a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1.63 \times 60}{0.85 \times 4 \times 90} = 0.32'' \not> h_f$$

$\because a \not> h_f$; It is a false T-beam, so singly reinforced beam design is needed.

3. Steel ratio:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} . \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.85 \times 0.85 \times \frac{4}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0181$$

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} = \frac{3 \times \sqrt{4000}}{60000} \geq \frac{200}{60000} = 0.0032 \geq 0.0033 \approx 0.0033$$

$\rho_{max} > \rho_{min}$ (ok)

ρ_{max} can be used.

4. Depth check:

$$M_u = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

$$\Rightarrow 2025 = 0.90 \times 0.0181 \times 60 \times 90 \times d^2 \left(1 - 0.59 \times 0.0181 \times \frac{60}{4}\right)$$

$\therefore d_{req.} = 5.24 \approx 6''$

$\therefore d_{provided} = 25.5\ in. > d_{req.}$ (ok)

5. Steel calculation:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$$

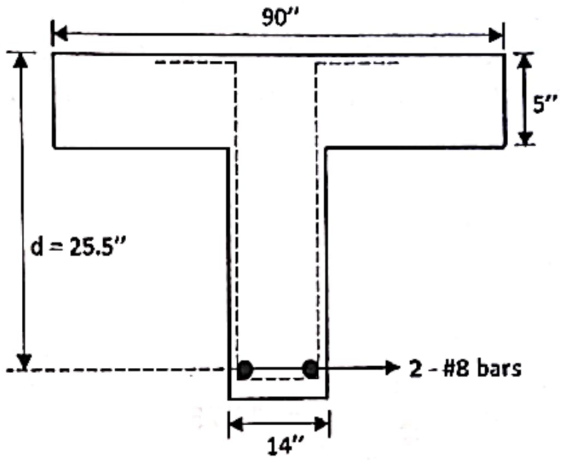
$$\therefore A_s = \frac{2025}{0.90 \times 60 \times \left(25.5 - \frac{a}{2}\right)} \text{ ------ } [1]$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$\therefore a = \frac{A_s \times 60}{0.85 \times 4 \times 90} \text{ ---------- } [2]$$

$$\therefore A_s = 1.48\ in^2$$

Provide 2 - #8 bars with $A_s = 1.58\ in^2$.



"You will not prosper until you spend what you love."

- Al Hadith

"Whatever you give up for fear of Allah,
Allah will surely give you something better than that."

- Al Hadith

Pb-18. Doubly Reinforced Beam: Analysis (Load determination) – *WSD:*
A simply supported beam 10×22.5 in. in cross section has a span of 19 ft. The tensile reinforcement consists of four No. 8 bars. Compressive reinforcement is provided by three No. 7 bars, the centroid of which is 2.5 in. from the compression face of the beam. If $f_s = 20{,}000\ psi, f_c = 1200\ psi$ & $n = 10$, what uniform live load per feet can the beam sustain?

Sol^n:
1. Design constants:

$n = 10$

$$r = \frac{f_s}{f_c} = \frac{20}{1.2} = 16.67$$

$$k = \frac{n}{n+r} = \frac{10}{10+16.67} = 0.375$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.375}{3} = 0.875$$

2. Check doubly/singly:

$Here, \quad A_s = 4 * 0.79 = 3.16\ in^2, \quad d = 22.5 - 3.5 = 19''$

$$\therefore M_1 = \frac{1}{2} f_c k j b d^2 = \frac{1}{2} \times 1.2 \times 0.375 \times 0.875 \times 10 \times 19^2 = 711\ k - in.$$

$Again, \quad M_1 = A_{s_1} f_s j d$

$$\therefore A_{s_1} = \frac{M_1}{f_s j d} = \frac{711}{20 \times 0.875 \times 20} = 2.03\ in^2$$

$A_s > A_{s_1}$; *so doubly* RC beam is required.

3. Moment:

$M_1 = 711\ k - in. \quad (in\ tension\ zone/bottom)$

$\therefore A_{s_2} = A_s - A_{s_1} = 3.16 - 2.03 = 1.13\ in^2$

$$\therefore f_s' = 2 f_s \frac{k - d'/d}{1-k} = 2 \times 20 \times \frac{0.375 - \frac{2.5}{19}}{1 - 0.375} = 15.58\ psi$$

$M_2 = A_s' f_s' (d - d')$

$\Rightarrow M_2 = (3 * 0.60) \times 15.58 \times (19 - 2.5)$

$\therefore M_2 = 463\ k - in. \quad (in\ comp.zone/top)$

$Again,\ M_2 = A_{s_2} f_s (d - d')$

$\therefore M_2 = 1.13 \times 20 \times (19 - 2.5) = 373$ k – in. [excess moment *in tension zone*]

$\because M_2 = 373\ k - in.$

$\therefore Moment\ capacity, M = M_1 + M_2 = 711 + 373 = 1084\ k - in. = 90333\ lb - ft.$

Self-weight of beam $= \frac{10 \times 22.5}{144} \times 150 = 234\ plf$

Live load $= w_L\ plf$

Total load, $w = (234 + w_L)\ plf$

$\therefore Moment,\ M = \frac{wl^2}{8}$

$\Rightarrow 90333 = \frac{(234 + w_L) \times (19)^2}{8}$

$\Rightarrow (234 + w_L) \times (19)^2 = 90333 \times 8$

$\therefore w_L = 2002 - 234 = 1768\ plf$

So, the beam can sustain uniform live load of 1768 lb/ft. or 1.77 k/ft. ***(Ans)***

2.6. Practice Problems:

Pb-19. Determine the concrete cross section and the steel area required for a simply supported rectangular beam with a span of 15 ft that is to carry a computed dead load of 1.3 k/ft. including the self-weight and a service live load of 2 k/ft. Assume $f_c' = 4\ ksi\ and\ f_y = 60\ ksi.$

Hints: Singly Reinforced Beam: Design – *USD* ($Ans: b = 10\ in., h = 18\ in.\ and\ A_s = 2.29\ in^2$)

Pb-20. A singly reinforced rectangular beam is to be designed, with effective depth approximately 1.5 times the width, to carry a service live load of 2000 lb/ft in addition to its own weight, on a 24 ft simple span. With $f_c' = 4000\ psi\ and\ f_y = 60000\ psi$, determine the required concrete dimensions b, d, h and steel reinforcing bars.

Hints: Singly Reinforced Beam: Design – *USD* ($Ans: b = 12\ in., d = 18\ in. h = 21\ in.\ and\ A_s = 3.68\ in^2, may\ use\ 3 - \#10\ bars$)

Pb-21. A simply supported rectangular beam has a total cross section of 10 × 16 in. and a length of 20 ft. It is reinforced with four No. 5 bars in one row. The distance from the centers of the bars to the lower surface of the beam is 2.5 in. With 2500 psi concrete and an allowable stress of 20,000 psi in the steel, what is the resisting moment in the beam?

Hints: Singly Reinforced Beam: Analysis – *WSD*

Pb-22. Design a 15 ft cantilever beam of uniform depth to carry a uniform dead load of 1.5 k/ft and a service live load of 2.4 k/ft. Assume a beam width b = 14 in., $f_c' = 4\ ksi\ and\ f_y = 60\ ksi.$

Hints: Singly Reinforced Beam: Design – *USD*

Pb-23. A rectangular concrete beam measures 12 in. wide and has an effective depth of 18 in. Compression steel consisting of two No. 8 bars is located 2.5 in. from the compression face of the beam and tensile steel consisting of three No. 10 bars in one layer. If $f_c' = 4000\ psi\ and\ f_y = 60000\ psi$, what is the design moment capacity of the beam?

Hints: Doubly Reinforced Beam: Analysis – *USD* ($Ans: M_u = 3240\ k - in. - approximately$)

Pb-24. A beam of 15 ft simple span carries a uniform live load of 600 plf and a concentrated load of 15,000 lb located at 5 ft from the left support. It is limited in size to an overall cross section of 10×18 in. What reinforcement must be used if $f_s = 20{,}000\ psi, f_c = 1800\ psi\ \&\ n = 8$? Assume that the tension steel will be placed in one row.

Hints: Doubly Reinforced Beam: Design – *WSD*

Pb-25. A beam is limited to the dimensions b = 15 in., d = 20 in. and d′ = 4 in. If M_D= 170 ft-k and M_L= 225 ft-k, select the reinforcing required. Assume $f_c' = 4\ ksi\ and\ f_y = 60\ ksi$.
Hints: Doubly Reinforced Beam: Design – *USD* ($Ans: A_s = 7.59\ in^2, A_s' = 3.14\ in^2$ – select bar)

Pb-26. The floor system consists of 3 in. slabs supported by 14 ft span beams spaced at 10 ft on centers. The beams have a web width of 14 in. and an effective depth of 18.5 in. Calculate the necessary reinforcement for a typical interior beam if the factored applied moment is 5080 k-in. Use $f_c' = 3\ ksi\ and\ f_y = 60\ ksi$.
Hints: T - Beam: Design – *USD* ($Ans: A_s = 5.56\ in^2, may\ use\ 6 - \#9\ bars\ in\ two\ rows$)

Pb-27. A series of concrete beams spaced at 7 ft on centers are acting on a simply supported span of 12 ft. The beam supports a reinforced concrete floor slab 3 in. thick. If the width of the web is 10 in. and d = 17 in.; the beam is reinforced with six No. 9 bars. Determine the moment strength of a typical interior beam. Assume $f_c' = 3.5\ ksi\ and\ f_y = 60\ ksi$.
Hints: T - Beam: Analysis – *USD* ($Ans: M_u = 4950\ k - in. - approximately$)

Pb-28. A floor slab 4 in. thick is supported by reinforced concrete beams 9 ft c/c which, together with the slab, act as T-beams. The beams are simply supported and their span is 19 ft. The slab supports a live load of 150 psf. The cross-section of each beam below the slab is 10 in. × 20 in.; the reinforcement consists of six No. 7 bars in two rows, 2 in. c/c vertically, the center of the lower row being 2.5 in. above the lower surface of the beam. Assuming n = 10, determine $f_s\ and\ f_c$.
Hints: T - Beam: Analysis (material strengths determination) – *WSD*

Chapter-3
Design of Slabs

3.1. Theory Analysis:

Q-1. Define one-way and two-way slab. What is the basic difference between them? What is the criteria for the two-way slab? *[Basic is always important]*

Ans: ***One-way slab:*** A slab which is supported by two parallel beams either side of the slab and the load is distributed perpendicular to the beams in a single direction is known as one-way slab.

Two-way slab: A slab which is supported by four beams around each sides of the slab and the load is distributed towards the beams in two directions is known as two-way slab.

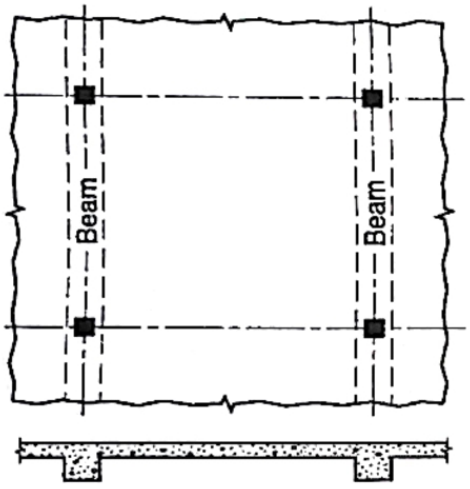


Fig: One-way slab

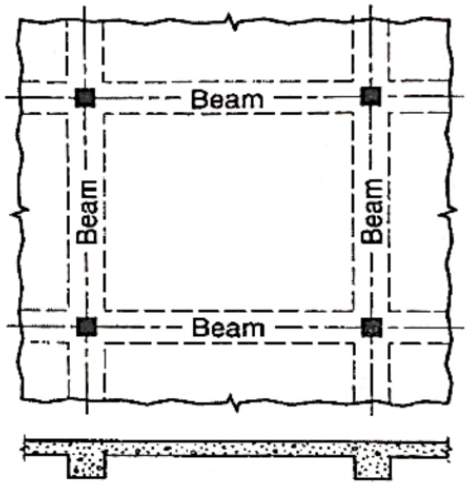


Fig: Two-way slab

Basic difference: The ratio of long span (l_b) to short span (l_a) is greater than or equal to 2 in the one-way slab.

Criteria for two-way slab:

i) $\frac{l_b}{l_a} < 2$

ii) It has supports around each sides of the slab.

iii) Load is transferred in two direction perpendicularly.

iv) Deflection occurs in both direction of the slab.

Q-2. Briefly describe the functional similarities between a one-way slab and a singly reinforced beam. [***CE-2018***]

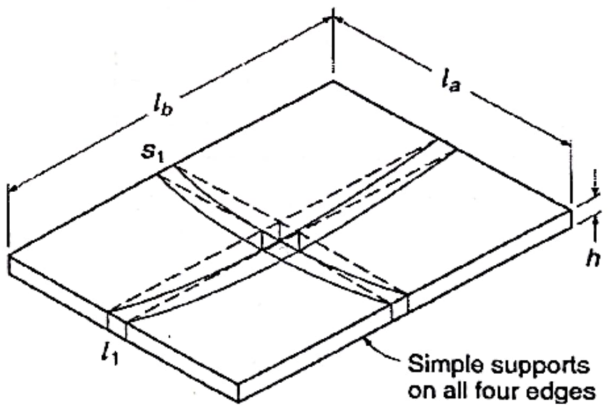
Ans: The functional similarities between a one-way slab and a singly reinforced beam are described below:

i) A one-way slab is assumed as a unit strip (12 in. wide) to be a rectangular beam.
ii) The thickness of one-way slab depends on bending, the deflection, and shear reinforcements as like a singly reinforced beam.
iii) Poisson's ratio is assumed to be zero.
iv) The load on the slab becomes the load per unit length on the slab strip like a beam.
v) The deflection is always neglected as the assumed for a singly reinforced rectangular beam.
vi) The flexure reinforcement of the slab is placed at right angles to its supporting edge beam as that is done in a rectangular beam.

Q-3. Prove that for a two-way slab the ratio of load carried by the slab in the direction of short span to the direction of long span is inversely proportional to the fourth power of the ratio of the both spans length.

Or, Show that, $\frac{W_a}{W_b} = \left(\frac{l_b}{l_a}\right)^4$

Ans: Let, for a two-way slab,
Short span length = l_a, Long span length = l_b
Uniform load = w, Flexural rigidity = EI
Deflection of short & long strips = δ_a & δ_b



The maximum deflection occurs at the midpoint is given by

$$\delta = \frac{5wl^4}{384\,EI}$$

In a two-way slab, the deflections at the intersection midpoint must be the same.
Thus, it is written that,

$$\delta_a = \delta_b$$

$$\Rightarrow \frac{5w_a {l_a}^4}{384\,EI} = \frac{5w_b {l_b}^4}{384\,EI}$$

$$\Rightarrow w_a {l_a}^4 = w_b {l_b}^4$$

[Proved]

Q-4. What are the key points should be followed for an inspection to observe the reinforcement placement of a slab before casting? [***CE-2018***]

Ans: The key points should be followed for an inspection to observe the reinforcement placement are as follows:

i) The reinforcement steel should be free from of any loose scale, mud or oil.
ii) The main reinforcement is placed in long direction.
iii) Distribution steel bar is placed orthogonally top of main bar in positive (+) moment region.
iv) Distribution steel bar is placed orthogonally bottom of main bar in negative (-) moment region.
v) The size & spacing of reinforcement should be observed as per drawing.
vi) Proper cover to reinforcement should be provided to the slab bottom.

Q-5. Why is it necessary to provide temperature and shrinkage reinforcement in one-way slab? [***CE-2016, 2014; BECM-2019***]

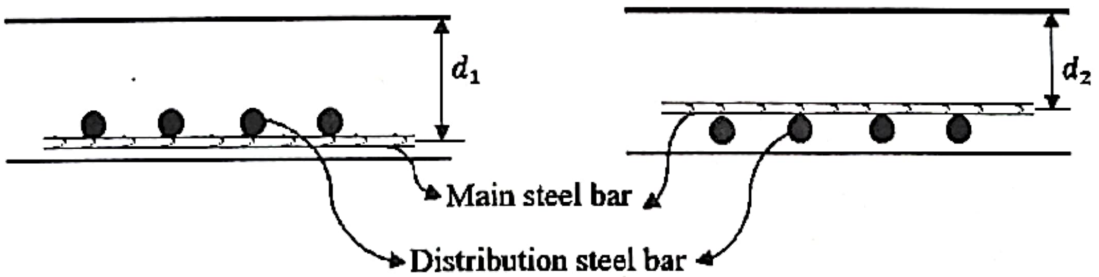
or, Why two-way slab does not require additional temperature and shrinkage reinforcement? [***CE-2017***]

Ans: In one-way slab, the reinforcement is provided for resisting the bending moments as well as to reduce shrinkage and distributing cracks. However, as contraction takes place equally in all directions, it is necessary to provide special reinforcement for shrinkage and temperature contraction in the direction perpendicular to the main reinforcement. This added steel is known as *temperature and shrinkage reinforcement* or *distribution steel.*

Q-6. Why is the distribution steel bar placed over the main steel bar in the one-way slab design?

Ans: The moment capacity of the steel bar is given by $M = A_s f_s j d$

Thus, the moment capacity (M) $\propto$ effective depth (d).



Here, $d_1 > d_2$ and so $M_1 > M_2$. If distribution bar is placed below the main bar, effective depth (d) is decreased and hence moment capacity of the main steel is also decreased.

So, for the increasing the moment capacity of steel bar, the distribution steel bar placed over the main steel bar in the one-way slab design.

Q-7. What is corner reinforcement? Why and where is it needed? How is it placed and what in amount? [***CE- 2019, 2015, 2014, 2013; BECM-2019, 2018, 2017***]

Ans: ***Corner reinforcement:*** The reinforcement which is provided to resist the torsional moment at the corner of two-way slab is known as the corner reinforcement.

Causes of need: In two-way slabs, the twisting moment at exterior corner is developed and causes crack in the slab at the bottom along the panel diagonal and at the top perpendicular to the panel diagonal.

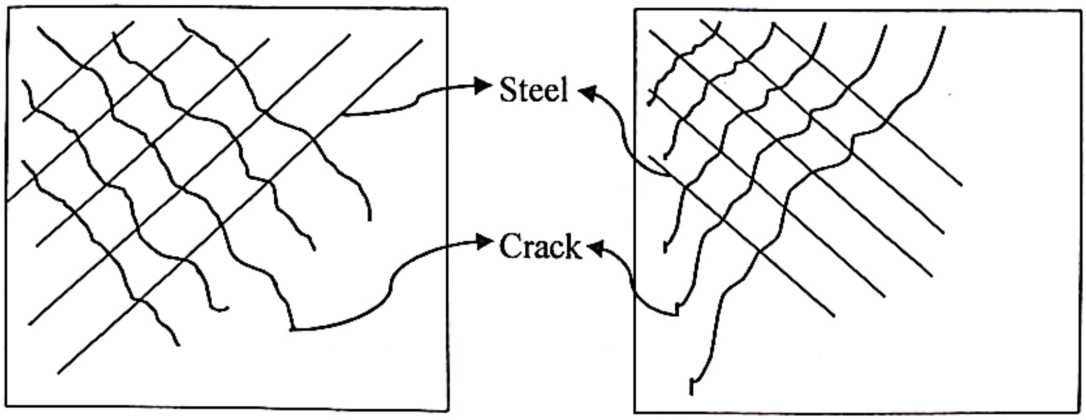
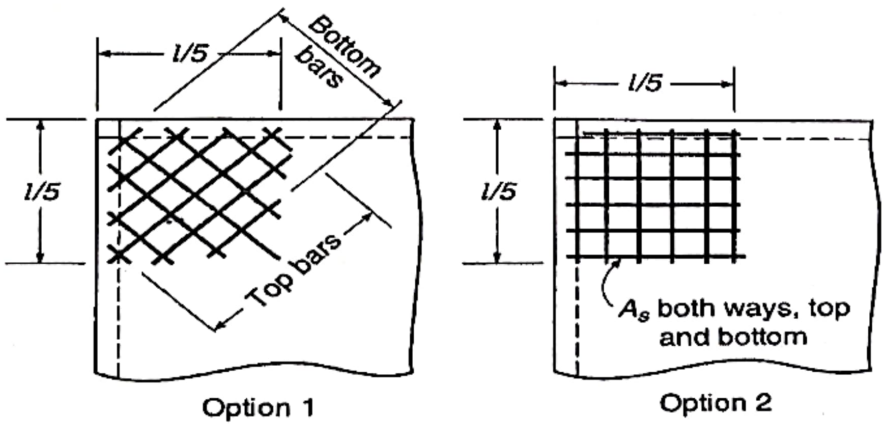


Fig: Cracks at bottom

Fig: Cracks at top

Placement & amount: Special reinforcement should be provided at exterior corners in both the bottom and top of the slab, for a distance in each direction from the corner equal to $\frac{1}{5}th$ of the longer span of the corner panel. The reinforcement at the top of the slab should be parallel to the diagonal from the corner and at the bottom should be perpendicular to the diagonal.

According to ACI code, the option (2) is adopted practically for the easy placement of steel bar.



3.2. Basic Concepts:

1. Minimum thickness of one-way slab:

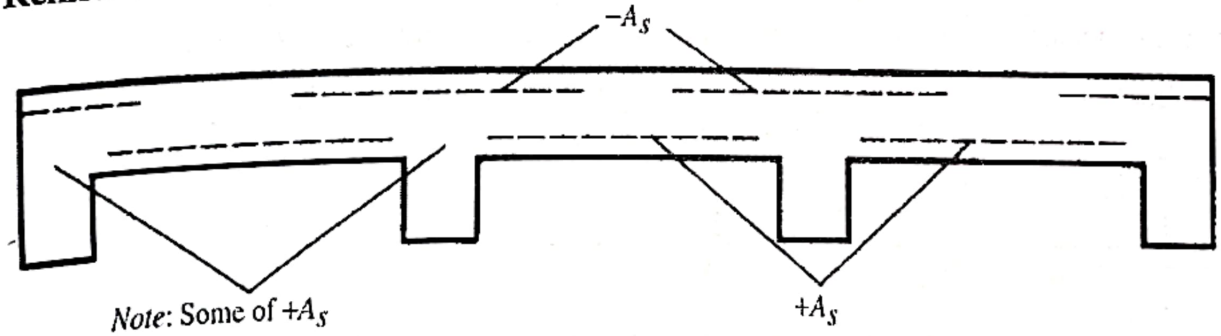
Support condition	Minimum thickness, $t_{min} \geq$	
	WSD	*USD*
Simply supported	$\frac{L}{25}$	$\frac{L}{20}$
One end continuous	$\frac{L}{30}$	$\frac{L}{24}$
Both ends continuous	$\frac{L}{35}$	$\frac{L}{28}$
Cantilever	$\frac{L}{12}$	$\frac{L}{10}$

*L = span length of a panel

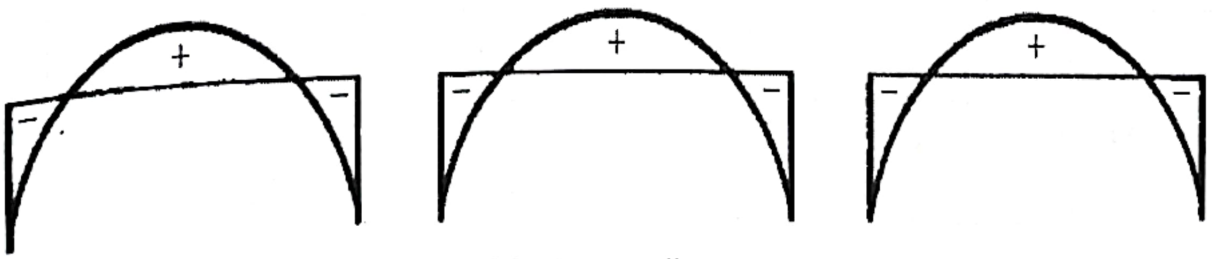
*In WSD 25, 30, 35 increasing by 5.

*In USD 20, 24, 28 increasing by 4. (Design is very easy)

2. Reinforcement details in continuous one-way slab:



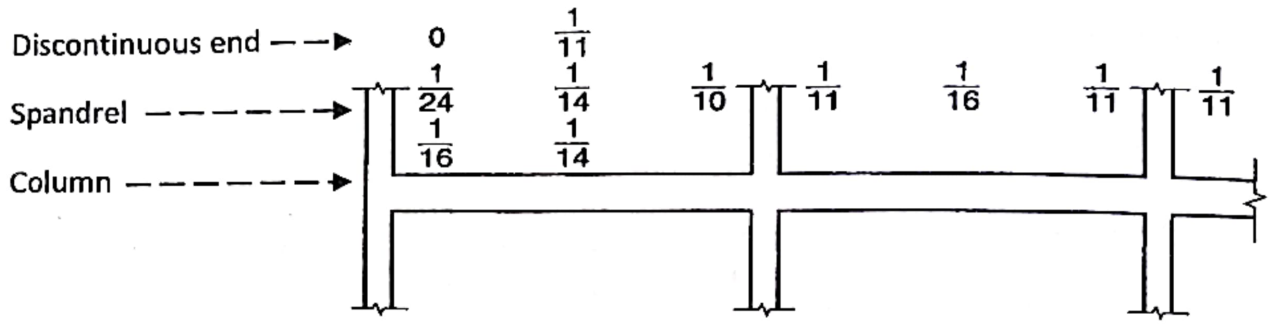
(a) Placement of bars



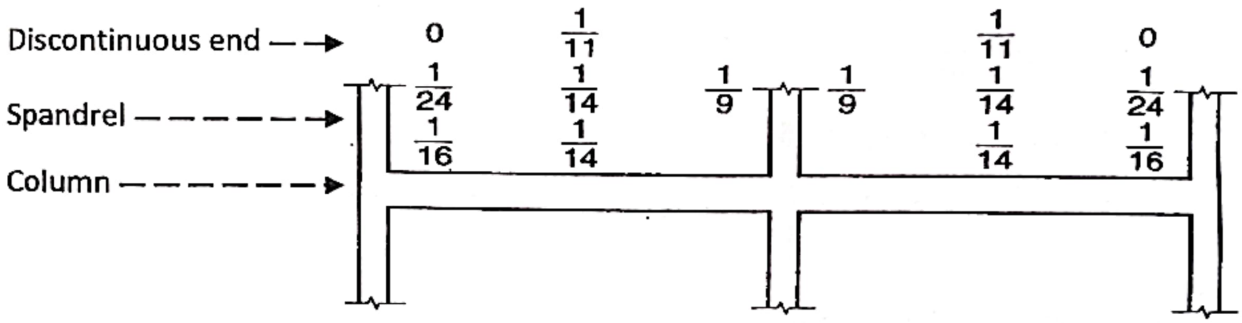
(b) Moment diagram

3. Moment coefficients for one-way slab design:

i) Slab is continuous over more than two spans:



ii) Slab is continuous over two spans only:



■ Listing the moments when slab is built **spandrel** / monolithically / integrally. ***(In exam, VVI)***

Section	Moment = coefficient * wL^2	
	More than 2 spans	*Only 2 spans*
At interior support, M(-ve) (first span)	$\frac{1}{10}$	$\frac{1}{9}$
At interior support, M(-ve) (interior span)	$\frac{1}{11}$	—
At midspan, M(+ve) (first span)	$\frac{1}{14}$	$\frac{1}{14}$
At midspan, M(+ve) (interior span)	$\frac{1}{16}$	—
At exterior support, M(-ve)	$\frac{1}{24}$	$\frac{1}{24}$

*Then maximum moment is taken in depth calculation.

3.3. Solution Techniques:

Design of one-way slab:

WSD

Step-1. Minimum thickness of slab, t_{min}

Step-2. Load calculation:
w = DL + LL + Floor finish

Step-3. Moment calculation:
Moment, M = coefficient * wL^2

Step-4. Design constant:

$$n = \frac{E_s}{E_c}$$

$$r = \frac{f_s}{f_c}$$

$$k = \frac{n}{n+r}$$

$$j = 1 - \frac{k}{3}$$

Step-5. Depth check:

$$d_{req.} = \sqrt{\frac{2M_{max}}{f_c kjb}}$$

Step-6. Steel calculation:

$$A_s = \frac{M}{f_s jd}$$

Step-7. Distribution steel:

$$A_{s(min)} = 0.0018bt$$

Step-8. Maximum spacing:
i) Main reinforcement, $S_{max} \leq 3t \; or \; 18''$
ii) Dist. reinforcement, $S_{max} \leq 5t \; or \; 18''$

Step-9. Reinforcement detailing:
Neat sketch

USD

Step-1. Minimum thickness of slab, t_{min}

Step-2. Load calculation:

w = 1.2DL + 1.6LL + 1.2 Floor finish

Step-3. Moment calculation:

Moment, M = coefficient * wL^2

Step-4. Maximum Reinforcement ratio:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}; \; \epsilon_t = 0.005$$

Step-5. Depth check:

$$M_{max} = \varphi \rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

Step-6. Steel calculation:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} \quad \boxed{1}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad \boxed{2}$$

Step-7. Distribution steel:

$$A_{s(min)} = 0.0018bt$$

Step-8. Maximum spacing:
i) Main reinforcement, $S_{max} \leq 3t \; or \; 18''$
ii) Dist. reinforcement, $S_{max} \leq 5t \; or \; 18''$

Step-9. Reinforcement detailing:
Neat sketch

3.4. Problem Solution:

3.4.1 Design of one-way slab - WSD

Pb-1: A reinforced concrete slab is built integrally with its supports and consists of ***two equal spans*** each with a clear span of 15 ft. The working live load is 100 psf, and 4000 psi concrete is specified for use with a yield stress of 60,000 psi. Design the slab using WSD method.

Sol^n:

1. Minimum thickness of slab,

$$t_{min} \geq \frac{L}{35} = \frac{15 \times 12}{35} = 5.14\ in. \approx 5.25\ in.$$

2. Load calculation:

i) Dead load $= \frac{5.25}{12} \times 150 = 66\ psf$

ii) Live load $= 100\ psf$

iii) Floor finish $= 30\ psf$

Total load, $w = 196\ psf$

3. Moment calculation:

At interior support, $M(-ve) = \frac{1}{9}wL^2 = \frac{1}{9} * 196 * 15^2 = 4900\ lb - ft$

At mid span, $M(+ve)\frac{1}{14}wL^2 = \frac{1}{14} * 196 * 15^2 = 3150\ lb - ft$

At exterior support, $M(+ve)\frac{1}{24}wL^2 = \frac{1}{24} * 196 * 15^2 = 1838\ lb - ft$

$\therefore M_{max} = 4900\ lb - ft$

4. Design constant:

$$Modular\ ratio, n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8$$

$$Stress\ ratio, r = \frac{f_s}{f_c} = \frac{0.40 \times 60{,}000}{0.45 \times 4000} = 13.33$$

$$k = \frac{n}{n+r} = 0.375$$

$$j = 1 - \frac{k}{3} = 0.875$$

5. Depth check:

$$d_{req.} = \sqrt{\frac{2M_{max}}{f_c kjb}} = \sqrt{\frac{2 \times 4900}{1800 \times 0.375 \times 0.875 \times 12}} = 4.1\ in.$$

$$\therefore d_{provided} = 5.25 - cc - \frac{Dia}{2}; \quad \left(cc + \frac{Dia}{2} = 1''\right)$$

$$= 5.25 - 0.75 - \frac{0.5}{2}$$

$$= 4.25\ in. > d_{req.} \quad (ok)$$

6. Steel calculation:

$$A_s = \frac{M}{f_s jd}$$

At interior support,

$$A_s = \frac{4900 \times 12}{24000 \times 0.875 \times 4.25} = 0.66\ in^2/ft$$

$$Spacing\ of\ \#3\ bars = \frac{12 \times 0.11}{0.66} = 2\ in.$$

$$Spacing\ of\ \#4\ bars = \frac{12 \times 0.20}{0.66} = 3.6\ in.$$

$$Provide\ \#4\ (12mm\ \varphi)\ bars\ @\ 3.5\ in.\ c/c$$

At mid span,

$$A_s = \frac{3150 \times 12}{24000 \times 0.875 \times 4.25} = 0.42\ in^2/ft$$

$$Provide\ \#4\ (12mm\ \varphi)\ bars\ @\ 5.5\ in.\ c/c$$

At exterior support,

$$A_s = \frac{1838 \times 12}{24000 \times 0.875 \times 4.25} = 0.24\ in^2/ft$$

$$Provide\ \#4\ (12mm\ \varphi)\ bars\ @\ 10\ in.\ c/c$$

7. Distribution steel:

$$A_{s(min)} = 0.0018bt = 0.0018 \times 12 \times 5.25 = 0.11\ in^2/ft$$

$Provide\ \#3\ (10mm\ \varphi)\ bars\ @\ 12\ in.\ c/c$, orthogonally to the direction of main reinforcement; at top of main steel in positive region and at bottom of main steel in negative region.

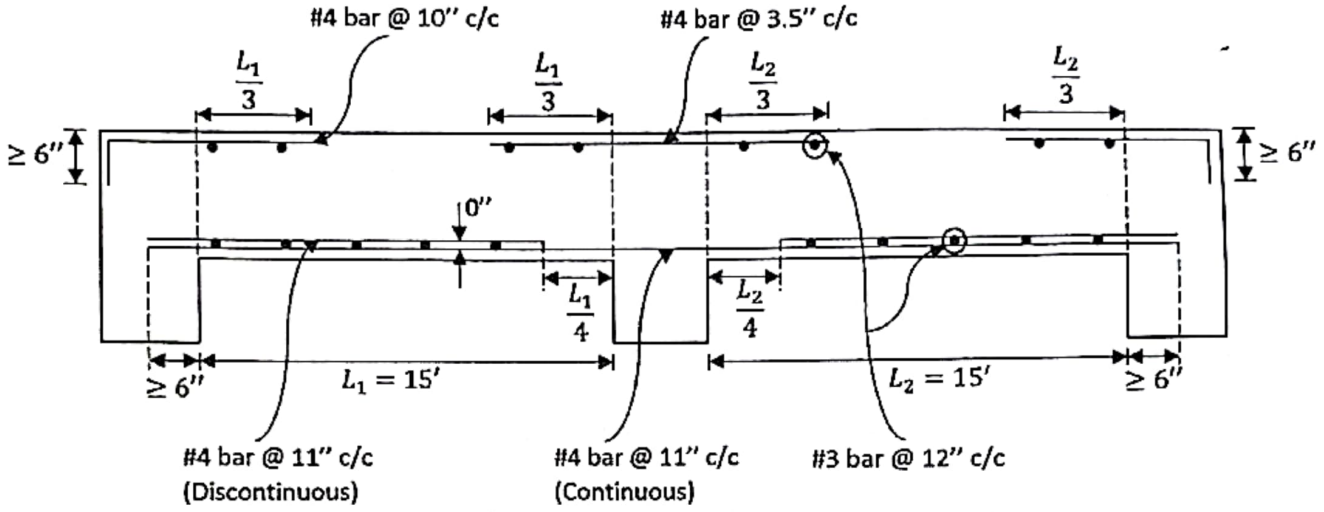
8. Maximum spacing:

i) Main reinforcement, $S_{max} \leq 3t \; or \; 18'' = 3 \times 5.25 = 15.75''$ (ok)

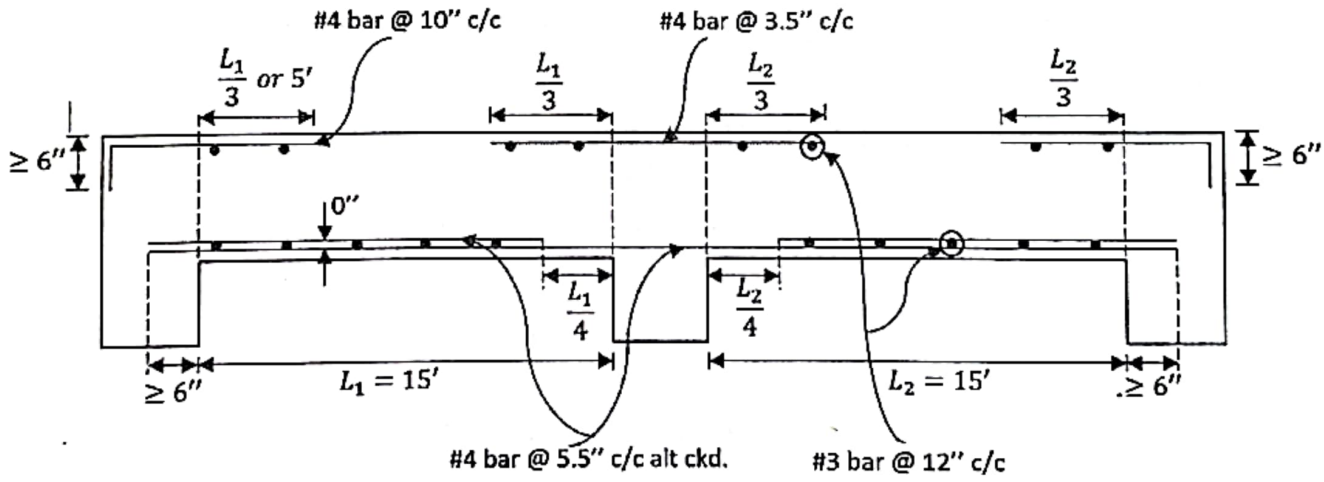
ii) Dist. reinforcement, $S_{max} \leq 5t \; or \; 18'' = 5 \times 5.25 = 26.25'' \approx 18''$ (ok)

9. Reinforcement detailing:

Straight bar arrangement



Or,



3.4.2 Design of one-way slab - $\boxed{USD}$

Pb-2: A reinforced concrete slab is built integrally with its supports and consists of ***two equal spans***, each with a clear span of 15 ft. The service live load is 100 psf, and 4000 psi concrete is specified for use with a yield stress of 60,000 psi. Design the slab using USD method. [***BECM-2019(sim)***]

Sol^n:

1. Minimum thickness of slab, $t_{min} \geq \frac{L}{28} = \frac{15\times12}{28} = 6.43\ in. \approx 6.50\ in.$

2. Load calculation:

i) Dead load $= \frac{6.50}{12} \times 150 = 81.25 \times 1.2 = 97.5\ psf$

ii) Live load $= 100 \times 1.6 = 160\ psf$

iii) Floor finish $= 30 \times 1.2 = 36\ psf$

Total load, $w = 293.5\ psf$

3. Moment calculation:

i) At interior support, $M(-ve)\frac{1}{9}wL^2 = \frac{1}{9} * 293.5 * 15^2 = 7.34\ k-ft$

ii) At mid span, $M(+ve)\frac{1}{14}wL^2 = \frac{1}{14} * 293.5 * 15^2 = 4.72\ k-ft$

iii) At exterior support, $M(-ve) = \frac{1}{24}wL^2 = \frac{1}{24} * 293.5 * 15^2 = 2.75\ k-ft$

$\therefore M_{max} = 7.34\ k-ft$

4. Maximum Reinforcement ratio,

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u+\epsilon_t}$$

$$\therefore \rho_{max} = 0.85 \times 0.85 \times \frac{4}{60} \times \frac{0.003}{0.003+0.005} = 0.0181$$

5. Depth check:

$$M_{max} = \varphi\rho f_y b d^2 \left(1 - 0.59\rho\frac{f_y}{f_c'}\right)$$

$$\Rightarrow 7.34 \times 12 = 0.90 \times 0.0181 \times 60 \times 12 \times d^2 \left(1 - 0.59 \times 0.0181 \times \frac{60}{4}\right)$$

$\therefore d_{req.} = 2.99\ in.$

$\because d_{provided} = 6.5 - 1 = 5.5\ in. > d_{req.} \quad (ok)$

6. Steel calculation:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} \quad \boxed{1}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad \text{——————} \boxed{2}$$

i) At interior support, $M_u = 7.34\ k-ft$
$\therefore A_s = 0.31\ in^2/ft$ *(by direct calculator)*
Provide #4 (12*mm* φ) *bars* @ 7.5 *in. c/c*

or, by iteration,

a	A_s	a
1	0.33	0.49
0.49	0.31	0.46
0.46	0.31	0.46

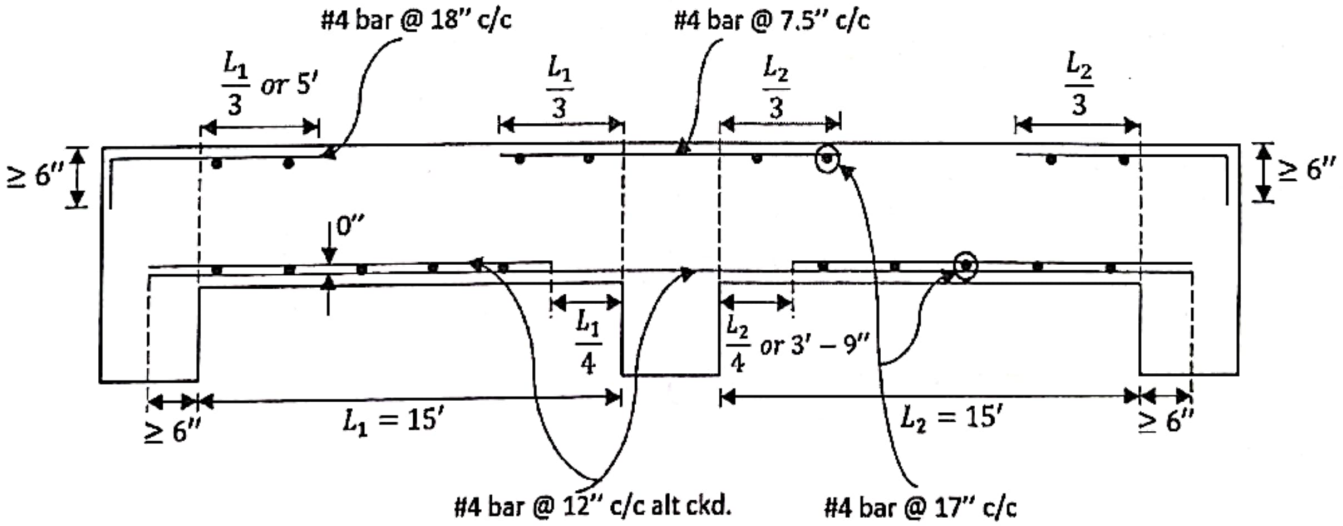
ii) At mid span, $M_u = 4.72\ k-ft$
$\therefore A_s = 0.20\ in^2/ft$
Provide #4 (12*mm* φ) *bars* @ 12 *in. c/c*

iii) At exterior support, $M_u = 2.75\ k-ft$
$\therefore A_s = 0.11\ in^2/ft$
Provide #4 (12*mm* φ) *bars* @ 21.8 18 *in. c/c*

7. Distribution steel:
$A_{s(min)} = 0.0018bt = 0.0018 \times 12 \times 6.5 = 0.14\ in^2/ft$
Provide #4 (12*mm* φ) *bars* @ 17 *in. c/c*

8. Maximum spacing:
i) Main reinforcement, $S_{max} \leq 3t\ or\ 18'' = 3 \times 6.5 = 19.5'' \approx 18''$ (ok)
ii) Dist. reinforcement, $S_{max} \leq 5t\ or\ 18'' = 5 \times 6.5 = 32.5'' \approx 18''$ (ok)

9. Reinforcement detailing:
Straight bar arrangement



Pb-3: A reinforced concrete one-way slab is continuous over ***three spans***, each having a clear span length of 20 ft. The slab is built monolithically with the supporting beam having 12 in. wide by 20 in. depth. Design the by strength design (*USD*) method and show reinforcement detailing. Use $f_c' = 3000\ psi\ and\ f_y = 60{,}000\ psi$. [***CE-2019***]

Sol^n:

1. Minimum thickness of slab, $t_{min} \geq \frac{L}{28} = \frac{20\times12}{28} = 8.57\ in. \approx 8.75\ in.$

2. Load calculation:

i) Dead load $= \frac{8.75}{12} \times 150 = 109.375 \times 1.2 = 131.25\ psf$

ii) Live load $= 100 \times 1.6 = 160\ psf$ (assume)

iii) Floor finish $= 30 \times 1.2 = 36\ psf$ (assume)

Total load, $w = 327.25\ psf$

3. Moment calculation:

i) At interior support (first span), $M(-ve) = \frac{1}{10}wL^2 = \frac{1}{10} * 327.25 * 20^2 = 13.1\ k-ft$

ii) At interior support (interior span), $M(-ve) = \frac{1}{11}wL^2 = \frac{1}{11} * 327.25 * 20^2 = 11.9\ k-ft$

iii) At mid span (first span), $M(+ve) = \frac{1}{14}wL^2 = \frac{1}{14} * 327.25 * 20^2 = 9.35\ k-ft$

iv) At mid span (interior span), $M(+ve) = \frac{1}{16}wL^2 = \frac{1}{16} * 327.25 * 20^2 = 8.18\ k-ft$

v) At exterior support, $M(-ve) = \frac{1}{24}wL^2 = \frac{1}{24} * 327.25 * 20^2 = 5.45\ k-ft$

$\therefore M_{max} = 13.1\ k-ft$

4. Maximum Reinforcement ratio:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$$

$$\therefore \rho_{max} = 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0135$$

5. Depth check:

$$M_{max} = \varphi\rho f_y b d^2 \left(1 - 0.59\rho\frac{f_y}{f_c'}\right)$$

$$\Rightarrow 13.1 \times 12 = 0.90 \times 0.0135 \times 60 \times 12 \times d^2 \left(1 - 0.59 \times 0.0135 \times \frac{60}{3}\right)$$

$\therefore d_{req.} = 4.62\ in.$

$\because d_{provided} = 8.75 - 1 = 7.75\ in. > d_{req.} \quad (ok)$

6. Steel calculation:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} \quad \boxed{1}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad \boxed{2}$$

i) At interior support (first span), $M_u = 13.1\ k-ft$
$\therefore A_s = 0.59\ in^2/ft$ $(by\ direct\ calculator)$
$Provide\ \#4\ (12mm\ \varphi)\ bars\ @\ 4\ in.c/c$

ii) At interior support (interior span), $M_u = 11.9\ k-ft$
$\therefore A_s = 0.53\ in^2/ft$
$Provide\ \#4\ (12mm\ \varphi)\ bars\ @\ 4.5\ in.c/c$

iii) At mid span (first span), $M_u = 9.35\ k-ft$
$\therefore A_s = 0.41\ in^2/ft$
$Provide\ \#4\ (12mm\ \varphi)\ bars\ @\ 5.5\ in.c/c$

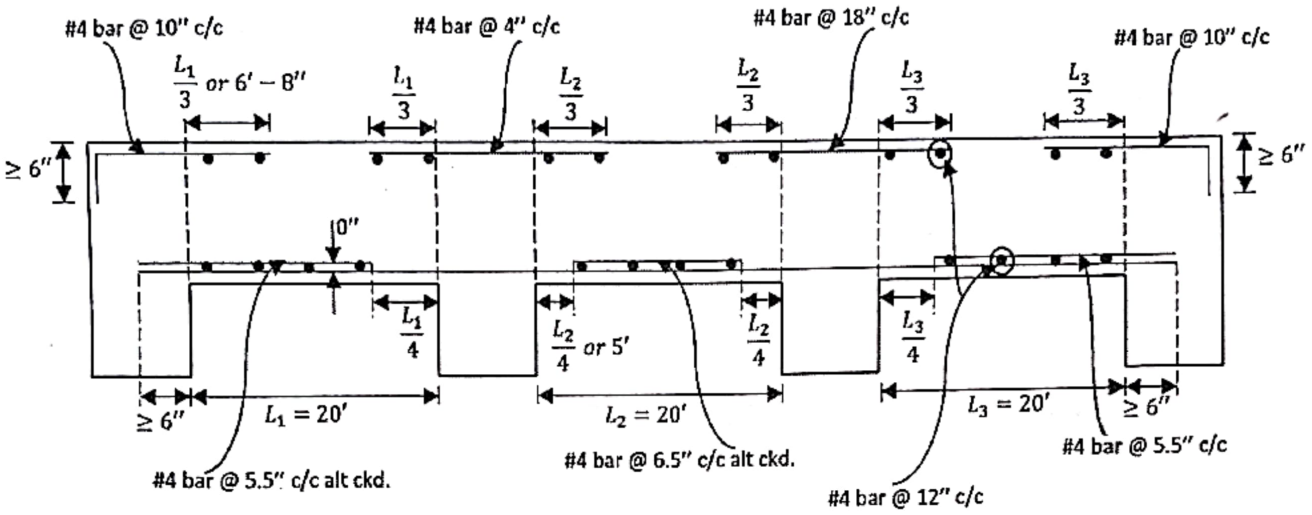
iv) At mid span (interior span), $M_u = 8.18\ k-ft$
$\therefore A_s = 0.35\ in^2/ft$
$Provide\ \#4\ (12mm\ \varphi)\ bars\ @\ 6.5\ in.c/c$

iii) At exterior support, $M_u = 5.45\ k-ft$
$\therefore A_s = 0.23\ in^2/ft$
$Provide\ \#4\ (12mm\ \varphi)\ bars\ @\ 10\ in.c/c$

7. Distribution steel:
$A_{s(min)} = 0.0018bt = 0.0018 \times 12 \times 8.75 = 0.19\ in^2/ft$
$Provide\ \#4\ (12mm\ \varphi)\ bars\ @\ 12\ in.c/c$

8. Maximum spacing:
i) Main reinforcement, $S_{max} \leq 3t\ or\ 18'' = 3 \times 8.75 = 26.25'' \approx 18''$ (ok)
ii) Dist. reinforcement, $S_{max} \leq 5t\ or\ 18'' = 5 \times 8.75 = 43.75'' \approx 18''$ (ok)

9. Reinforcement detailing:
Straight bar arrangement



Pb-4: A one-way slab is continuous over ***five equal spans***, length of 15 ft. each. The slab has to carry a live load of 60 psf, floor finish of 30 psf and its own weight. Design the by *USD* method. Use $f_c' = 3000\ psi$ *and* $f_y = 60{,}000\ psi$. [***CE-2018***]

Soln:

1. Minimum thickness of slab, $t_{min} \geq \frac{L}{28} = \frac{15\times12}{28} = 6.43\ in. \approx 6.50\ in.$

2. Load calculation:

i) Dead load $= \frac{6.50}{12} \times 150 = 81.25 \times 1.2 = 97.5\ psf$
ii) Live load $= 60 \times 1.6 = 96\ psf$
iii) Floor finish $= 30 \times 1.2 = 36\ psf$
Total load, $w = 229.5\ psf$

3. Moment calculation:

i) At interior support (first span), $M(-ve) = \frac{1}{10}wL^2 = \frac{1}{10} * 229.5 * 15^2 = 5.16\ k-ft$

ii) At interior support (interior span), $M(-ve) = \frac{1}{11}wL^2 = \frac{1}{11} * 229.5 * 15^2 = 4.69\ k-ft$

iii) At mid span (first span), $M(+ve) = \frac{1}{14}wL^2 = \frac{1}{14} * 229.5 * 15^2 = 3.69\ k-ft$

iv) At mid span (interior span), $M(+ve) = \frac{1}{16}wL^2 = \frac{1}{16} * 229.5 * 15^2 = 3.23\ k-ft$

v) At exterior support, $M(+ve) = \frac{1}{24}wL^2 = \frac{1}{24} * 229.5 * 15^2 = 2.15\ k-ft$

$$\therefore M_{max} = 5.16\ k-ft$$

4. Maximum Reinforcement ratio,

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$$

$$\therefore \rho_{max} = 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.005} = 0.0135$$

5. Depth check:

$$M_{max} = \varphi\rho f_y b d^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

$$\Rightarrow 5.16 \times 12 = 0.90 \times 0.0135 \times 60 \times 12 \times d^2 \left(1 - 0.59 \times 0.0135 \times \frac{60}{3}\right)$$

$$d_{req.} = 2.90\ in.$$

$$\therefore d_{provided} = 6.5 - 1 = 5.5\ in. > d_{req.} \qquad (ok)$$

6. Steel calculation:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} \quad \boxed{1}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad \boxed{2}$$

i) At interior support (first span), $M_u = 5.16\ k-ft$

$\therefore A_s = 0.22\ in^2/ft$ *(by direct calculator)*

Provide #4 (12mm φ) bars @ 10.5 in. c/c

ii) At interior support (interior span), $M_u = 4.69\ k-ft$

$$\therefore A_s = 0.196\ in^2/ft$$

Provide #4 (12mm φ) bars @ 12 in. c/c

iii) At mid span (first span), $M_u = 3.69\ k-ft$
$\therefore A_s = 0.15\ in^2/ft$
Provide #4 $(12mm\ \varphi)$ *bars* @ 16 $in.\ c/c$

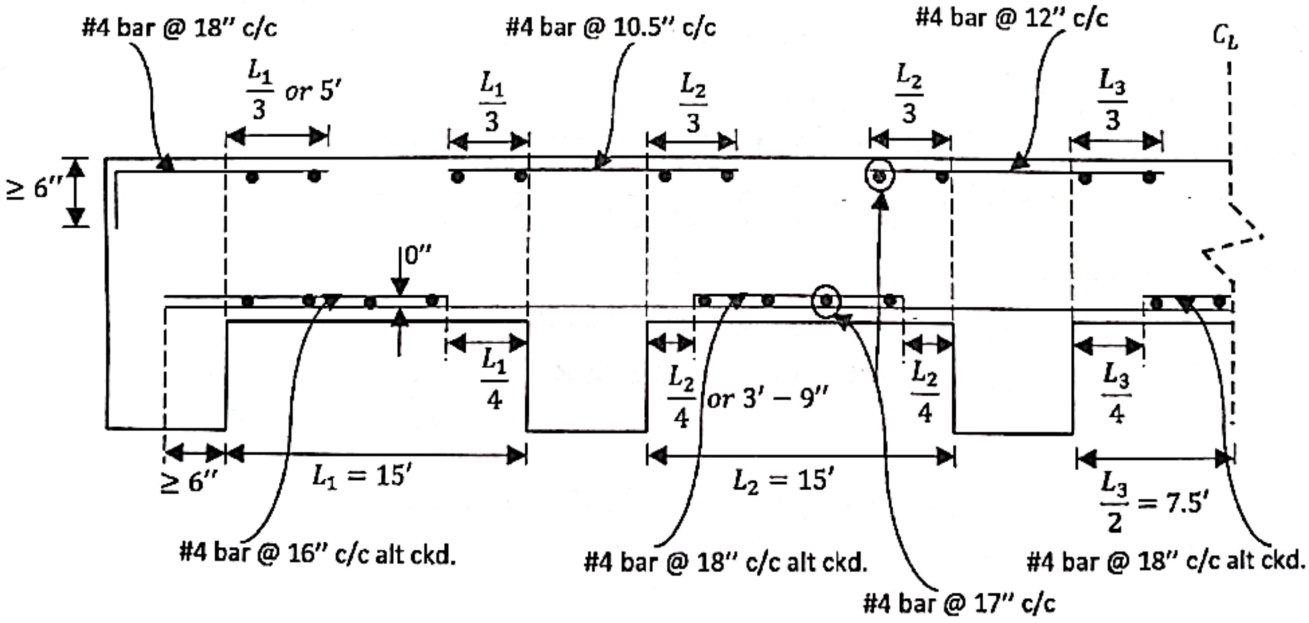
iv) At mid span (interior span), $M_u = 3.23\ k-ft$
$\therefore A_s = 0.13\ in^2/ft$
Provide #4 $(12mm\ \varphi)$ *bars* @ 18 $in.\ c/c$

iii) At exterior support, $M_u = 2.15\ k-ft$
$\therefore A_s = 0.088\ in^2/ft$
Provide #4 $(12mm\ \varphi)$ *bars* @ 27 18 $in.\ c/c$

7. Distribution steel:
$A_{s(min)} = 0.0018bt = 0.0018 \times 12 \times 6.5 = 0.14\ in^2/ft$
Provide #4 $(12mm\ \varphi)$ *bars* @ 17 $in.\ c/c$

8. Maximum spacing:
i) Main reinforcement, $S_{max} \leq 3t\ or\ 18'' = 3 \times 6.5 = 19.5'' \approx 18''$ (ok)
ii) Dist. reinforcement, $S_{max} \leq 5t\ or\ 18'' = 5 \times 6.5 = 32.5'' \approx 18''$ (ok)

9. Reinforcement detailing:
Straight bar arrangement



3.4.3 Design of two-way slab - *USD – ACI coefficient method*

Pb-5. Design of corner panel: A monolithic reinforced concrete slab consists of rectangular bays measuring 22× 24 ft. and width of the supporting beam is 16 in. The slab has to carry a live load of 100 psf in addition to its own weight. Design the corner panel of that slab system by ACI coefficient method and show reinforcement detailing. Use $f_c' = 4000\ psi$ *and* $f_y = 60{,}000\ psi$. [***CE-2019(sim); BECM-2019(sim), 2017***]

Sol^n:

Width of beam = 16 in.

Clear short span, $l_a = 22 - \frac{16}{12} = 20.67\ ft$

Clear long span, $l_a = 24 - \frac{16}{12} = 22.67\ ft$

Minimum slab thickness, $h = \frac{perimeter}{180} = \frac{2(20.7+22.67)\times 12}{180} = 5.78\ in. \approx 6\ in.$

2. Load calculation:

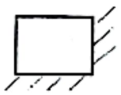
i) Dead load $= \frac{6}{12} \times 150 = 75 \times 1.2 = 90\ psf$

ii) Live load $= 100 \times 1.6 = 160\ psf$

Total load, $w = 250\ psf$

3. Panel ratio: $m = \frac{l_a}{l_b} = \frac{20.67}{22.67} = 0.91$

4. Moment calculation: (*corner panel has 2 continuous direction*)



i) Negative moment at continuous edge,

$M_{a(-ve)} = C_{a(-ve)} * w * {l_a}^2 = 0.059 \times 250 \times 20.67^2 = 6302\ lb-ft = 75{,}624\ lb-in.$

$M_{b(-ve)} = C_{b(-ve)} * w * {l_b}^2 = 0.041 \times 250 \times 22.67^2 = 5268\ lb-ft = 63{,}216\ lb-in.$

ii) Positive moments,

$M_{a(+ve)DL} = C_{a(+ve)DL} * w_{DL} * {l_a}^2 = 0.0324 \times 90 \times 20.67^2 = 14{,}950\ lb-in.$

$M_{a(+ve)LL} = C_{a(+ve)LL} * w_{LL} * {l_a}^2 = 0.0382 \times 160 \times 20.67^2 = 31{,}336\ lb-in.$

$M_{a(+ve)} = 46{,}286\ lb-in.$

$M_{b(+ve)DL} = C_{b(+ve)DL} * w_{DL} * {l_b}^2 = 0.0224 \times 90 \times 22.67^2 = 12{,}433\ lb-in.$

$M_{b(+ve)LL} = C_{b(+ve)LL} * w_{LL} * {l_b}^2 = 0.059 \times 160 \times 22.67^2 = 26{,}247\ lb-in.$

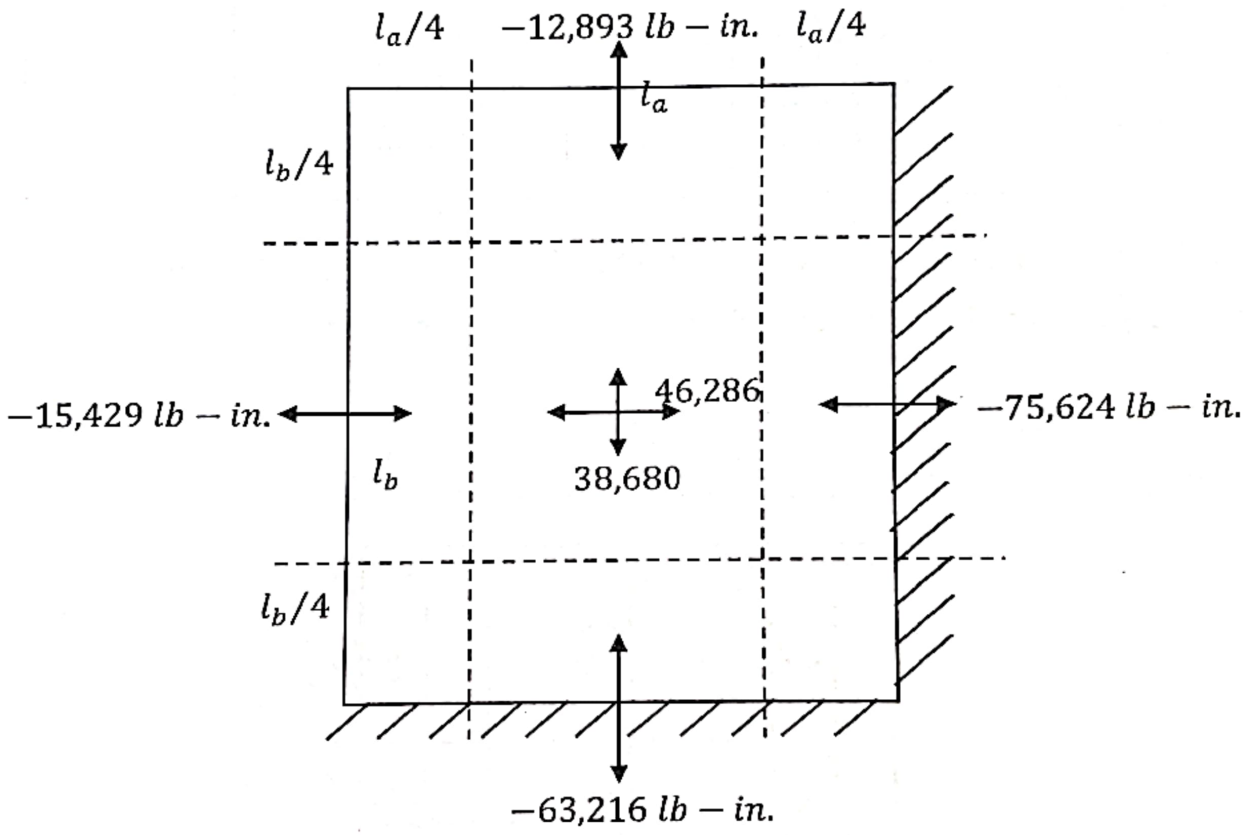
$M_{b(+ve)} = 38{,}680\ lb-in.$

iii) Negative moment at discontinuous edges,

$= \frac{1}{3} rd\ of\ the\ corresponding\ positive\ moments$

$\therefore M_{a(-ve)dce} = \frac{1}{3} \times 46{,}286 = 15{,}429\ lb - in.$

$\therefore M_{b(-ve)dce} = \frac{1}{3} \times 38{,}680 = 12{,}893\ lb - in.$



5. Steel calculation:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} \quad \boxed{1}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad \boxed{2}$$

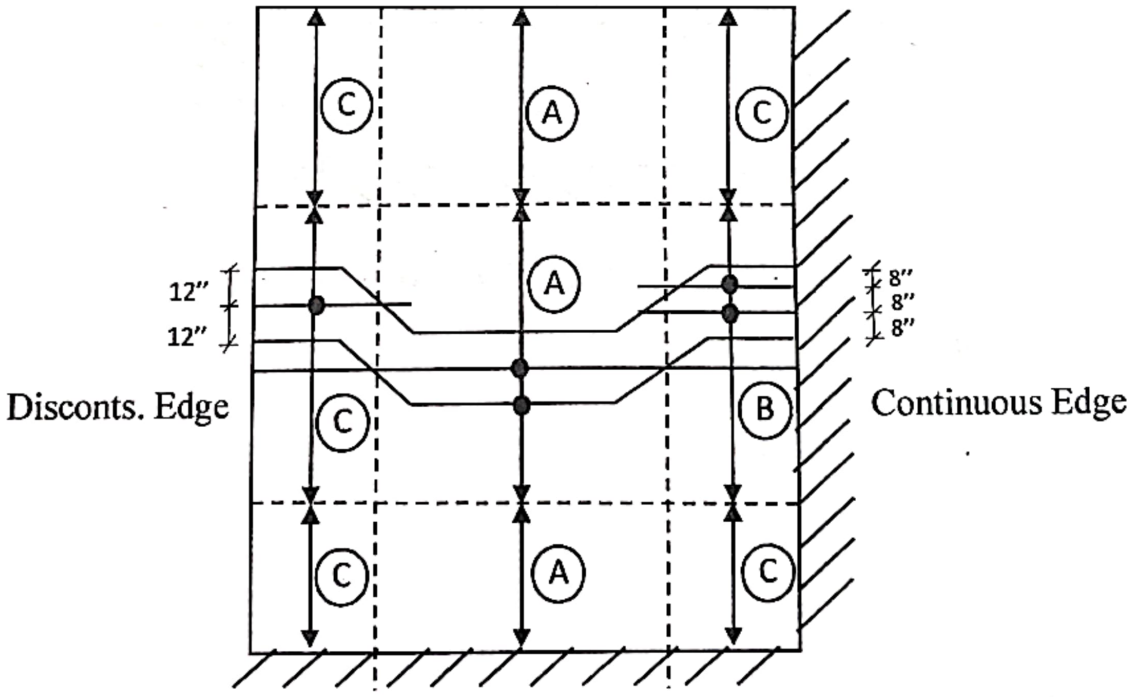
Here, $d_{provided} = t - cc = 6 - 1 = 5\ in.$

Location	*Moment (lb-in.)*	A_s (in^2)	*Spacing*	*Max. spacing*
Short direction				
-Middle strip				
--Mid span	46,286	0.176	#4 bar @ 12 in. c/c	$2t$ *or* 12 *in.*
--Continuous edge	75,623	0.293	#4 bar @ 8 in. c/c	
--Disconts. edge	15,429	0.057	#4 bar @ 12 in. c/c	
Long direction				
-Middle strip				
--Mid span	38,680	0.146	#3 bar @ 8 in. c/c	
--Continuous edge	63,213	0.243	#3 bar @ 4 in. c/c	
--Disconts. edge	12,893	.048	#3 bar @ 12 in. c/c	

For column strip, moments are $\frac{2}{3}$ times of that of middle strip. So, spacing of the same bar would be $\frac{3}{2}$ times of that of middle strip but should not exceed 12 in.

6. Reinforcement detailing:

i) Short direction:



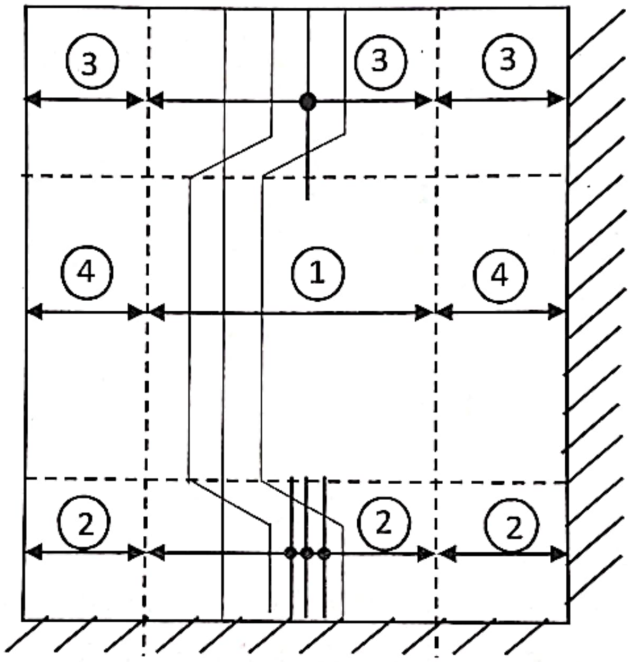
Legends:

A = #4 bar @ 12 in. c/c alt ckd

B = 2 - #4 extra top in bet^{n} two ckd

C = 1 - #4 extra top in bet^{n} two ckd

i) Long direction:



Legends:

(1) = #3 bar @ 8 in. c/c alt ckd

(2) = 3 - #3 extra top in bet^{n} two ckd

(3) = 1 - #3 extra top in bet^{n} two ckd

(4) = #3 bar @ 12 in. c/c alt ckd

Pb-6. Design of interior panel: A reinforced concrete floor consists of rectangular bays measuring 21× 26 ft. c/c on supporting beams on all four sides. The floor has to carry standard live load for a classroom, appropriate floor finish and its own dead weight. Design a typical interior panel of the slab system by ACI coefficient method and show reinforcement detailing. Use $f_c' = 4000\ psi$ *and* $f_y = 60{,}000\ psi$. [*CE-2018*]

Sol^n:

Clear short span, $l_a = 21\ ft$

Clear long span, $l_a = 26\ ft$

Minimum slab thickness, $h = \frac{perimeter}{180} = \frac{2(21+26)\times 12}{180} = 6.27\ in. \approx 6.50\ in.$

2. Load calculation:

i) Dead load $= \frac{6.5}{12} \times 150 = 81.25 \times 1.2 = 97.5\ psf$

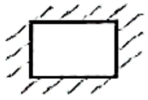
ii) Live load $= 40 \times 1.6 = 64\ psf\ \ (for\ classroom)$

iii) Floor finish $= 30 \times 1.2 = 36\ psf$

Total load, $w = 197.5\ psf$

3. Panel ratio: $m = \frac{l_a}{l_b} = \frac{21}{26} = 0.81$

4. Moment calculation: (*interior panel has 4 continuous direction*)



i) Negative moment at continuous edge,

$M_{a(-ve)} = C_{a(-ve)} * w * {l_a}^2 = 0.064 \times 197.5 \times 21^2 = 5574\ lb-ft = 66{,}888\ lb-in.$

$M_{b(-ve)} = C_{b(-ve)} * w * {l_b}^2 = 0.028 \times 197.5 \times 26^2 = 3738\ lb-ft = 44{,}856\ lb-in.$

ii) Positive moments,

$M_{a(+ve)DL} = C_{a(+ve)DL} * w_{DL} * {l_a}^2 = 0.0256 \times (97.5+36) \times 21^2 = 18{,}086\ lb-in.$

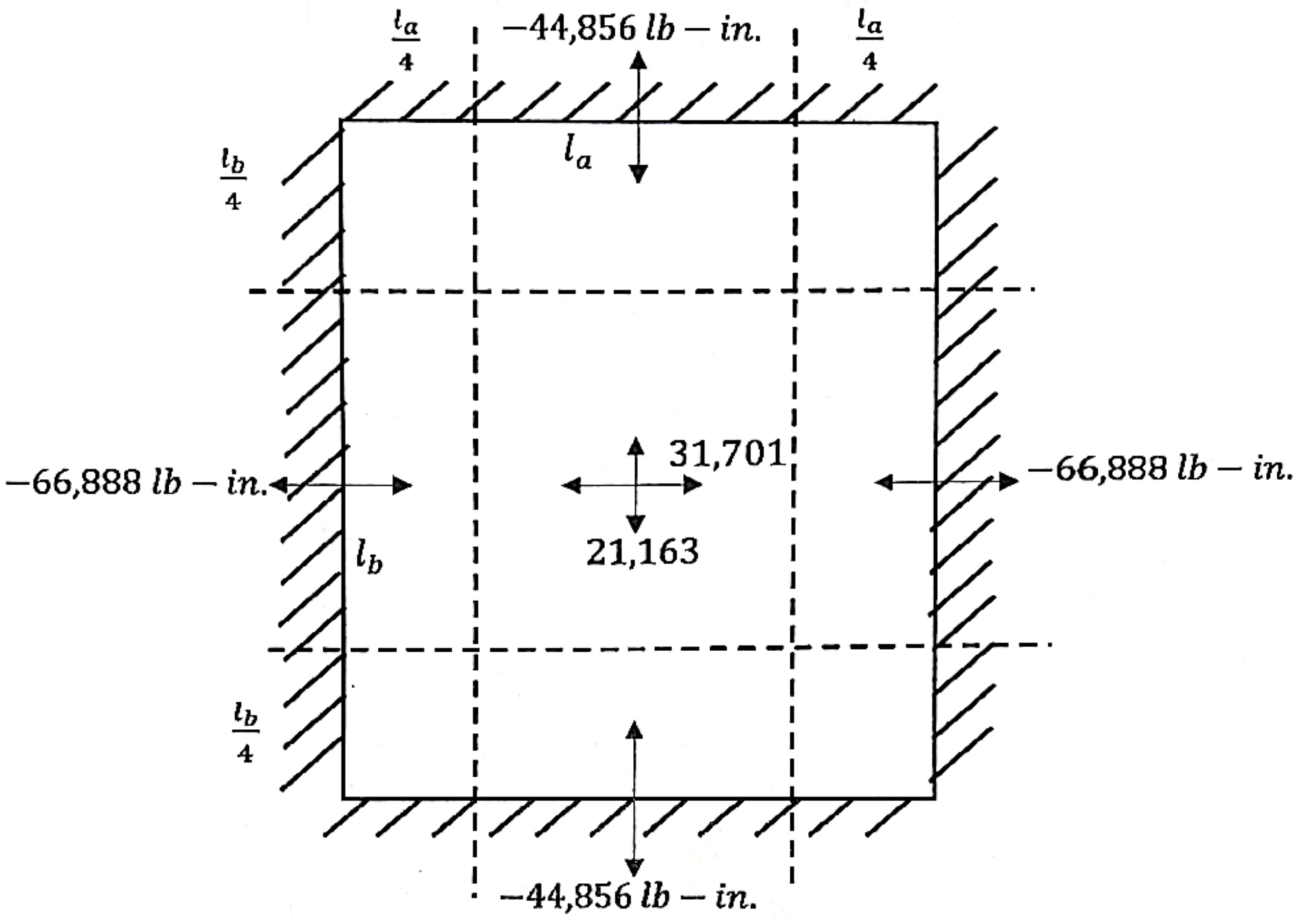
$M_{a(+ve)LL} = C_{a(+ve)LL} * w_{LL} * {l_a}^2 = 0.0402 \times 64 \times 21^2 = 13{,}615\ lb-in.$

$M_{a(+ve)} = 31{,}701\ lb-in.$

$M_{b(+ve)DL} = C_{b(+ve)DL} * w_{DL} * {l_b}^2 = 0.0112 \times (97.5+36) \times 26^2 = 12{,}129\ lb-in.$

$M_{b(+ve)LL} = C_{b(+ve)LL} * w_{LL} * {l_b}^2 = 0.0174 \times 64 \times 26^2 = 9{,}034\ lb-in.$

$M_{b(+ve)} = 21{,}163\ lb-in.$



5. Steel calculation:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} \quad \boxed{1}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad \boxed{2}$$

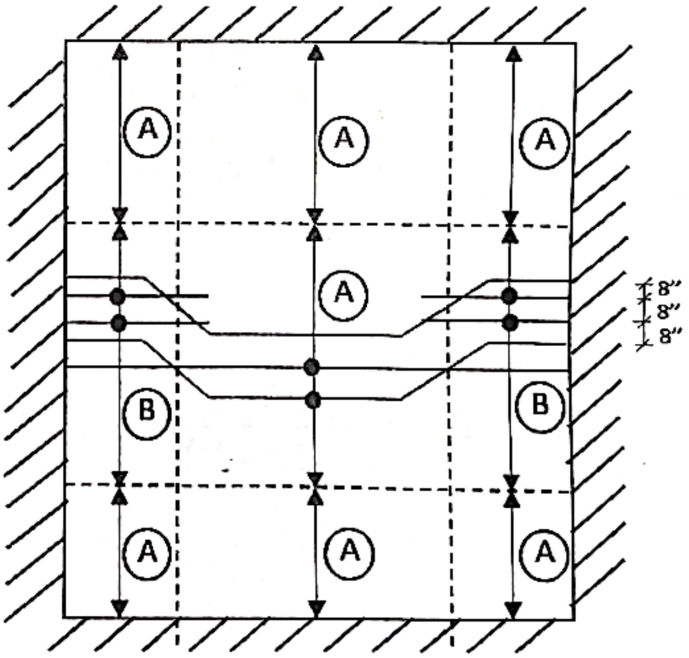
Here, $d_{provided} = t - cc = 6.50 - 1 = 5.50\ in.$

Location	*Moment (lb-in.)*	A_s (in^2)	*Spacing*	*Max. spacing*
Short direction				
-Middle strip				
--Mid span	31,701	0.108	#4 bar @ 12 in. c/c	$2t$ *or* 12 *in.*
--Continuous edge	66,888	0.232	#4 bar @ 8 in. c/c	
Long direction				
-Middle strip				
--Mid span	21,163	0.072	#3 bar @ 12 in. c/c	
--Continuous edge	44,856	0.154	#3 bar @ 8 in. c/c	

For column strip, moments are $\frac{2}{3}$ times of that of middle strip. So, spacing of the same bar would be $\frac{3}{2}$ times of that of middle strip but should not exceed 12 in.

6. Reinforcement detailing:

(i) Short direction:

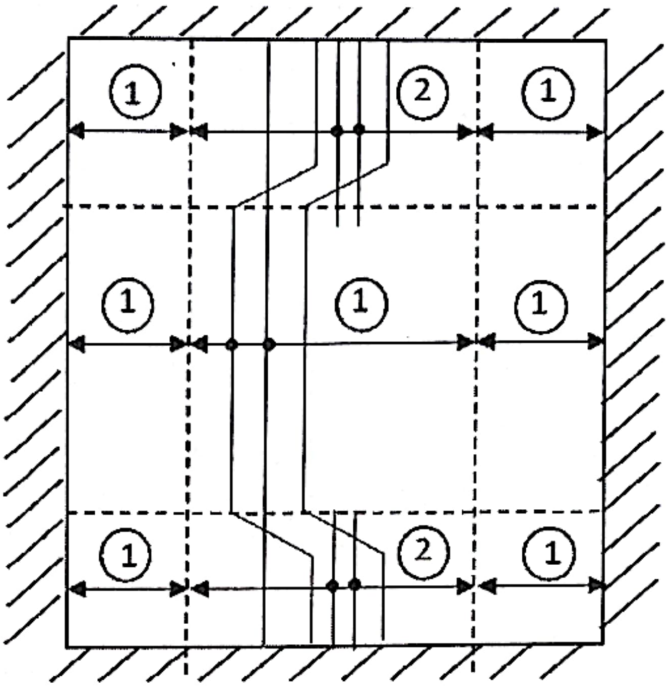


Legends:

(A) = #4 bar @ 12 in. c/c alt ckd

(B) = 2 - #4 extra top in bet^n two ckd

(ii) Long direction:



Legends:

(1) = #3 bar @ 12 in. c/c alt ckd

(2) = 2 - #3 extra top in betⁿ two ckd

3.5 Practice Problem:

Pb-7. Design a one-way slab continuous over integral beams and spanning 12 ft between supports. It carries 100 psf live load, and appropriate floor finish in addition to its own weight. Material strengths are $f_c' = 3000\ psi$ *and* $f_y = 60{,}000\ psi$. Design the slab by strength design method and show reinforcing detailing for main reinforcement and distribution reinforcement. [***CE-2015***]

Pb-8. The roof slab of a lobby having clear dimensions of 12 ft by 25 ft is supported by beams on all sides. Design the slab by strength design method using $f_c' = 4000\ psi$ *and* $f_y = 60{,}000\ psi$. Show reinforcement detailing in a cross section. [***CE-2014***]

Pb-9. A one-way solid slab spanning 16 ft between masonry walls carries a service live load of 100 psf and a 2 kips concentrated load, assumed to be uniformly distributed across the slab span. A 2 in. floor finish will be used, weighing 20 psf. Prepare a design for the slab, using $f_c' = 4000\ psi$ *and* $f_y = 60{,}000\ psi$. Show all concrete dimensions and reinforcement detailing on a simple sketch. [***CE-2013***]

Pb-10. The corner panel of an edge-supported reinforced concrete slab measures 16 ft × 20 ft between supports. The slab has to carry a live load of 100 psf, floor finish of 30 psf and its own weight. Design the slab by the coefficient method. Use $f_c' = 3000\ psi$ *and* $f_y = 60{,}000\ psi$. [***CE-2019***]

Pb-11. A two way concrete slab roof is to be designed to cover a transformer vault. The outside dimensions of the vault are 18 ft × 22 ft and supporting walls are 12 in. thick masonry. A service live load of 80 psf distributed uniformly over the roof surface will be assumed, and a dead load allowance of 10 psf added to the self-weight of the slab. Design the roof as a two-way edge-supported slab. Use $f_c' = 4000\ psi$ *and* $f_y = 50{,}000\ psi$. [***CE-2016***]

Pb-12. A monolithic reinforced concrete floor consists of rectangular bays measuring 21 × 26 ft. and the width of the supporting beam is 12 in. The floor is intended to carry a service live load of 125 psf uniformly distributed over its surface in addition to its self-weight. Design a typical interior panel and show reinforcement detailing. Use $f_c' = 3000\ psi$ *and* $f_y = 60{,}000\ psi$. [***CE-2015***]

Chapter-4
Shear and Diagonal Tension in Beams

4.1. Theory Analysis:

Q-1. What is meant by diagonal tension? Describe diagonal tension cracking in beams with appropriate sketch. [*CE-2014, 2013; BECM-2018*]

Ans: ***Diagonal tension***: The tension which is caused in the tensile zone of the beam due to shear, at or near the supports is called as diagonal tension.

Diagonal tension cracking in beams:

i) *Web-shear crack:* An inclined crack will develop independently in a beam, even though no flexure cracks are in that locality. Such cracks, which are called web–shear cracks. They also sometimes occur near the points of inflection of continuous beams or near simple supports.

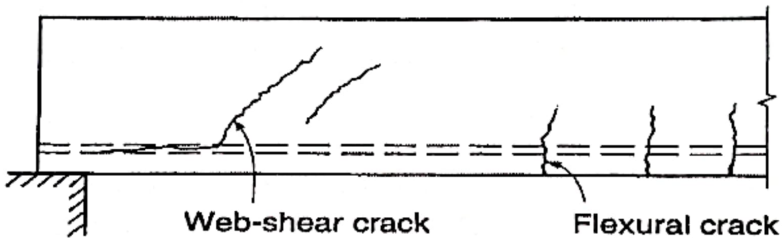


Fig: Web-shear crack

i) *Flexure-shear crack:* When inclined cracks can develop in the webs of reinforced concrete beams as well as extensions of flexural cracks, it is known as flexure-shear crack. For occurring cracks, the moment must be larger than the cracking moment, and the shear must be rather large. The cracks run at angles of about 45° with the beam axis and probably start at the top of a flexure crack.

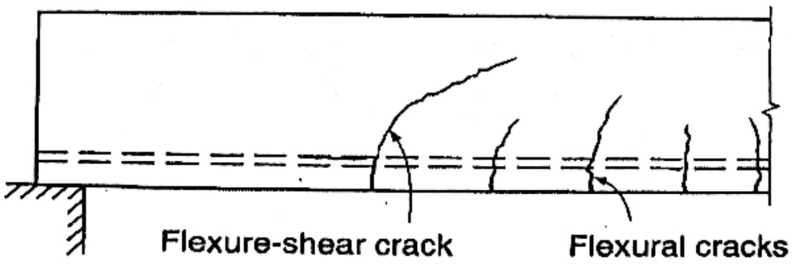


Fig: Flexure-shear crack

Q-2. Which type of beam failure is more dangerous (flexure failure or shear failure)? Justify your answer. [*CE-2014*]
Or, Why strength reduction factor φ is more conservative in shear reinforcement design for beams? [*CE-2013*]
Ans: Shear failure is more dangerous than flexure failure. The steel is provided in flexure crack zone but there is no steel in shear crack zone. Again only concrete has to carry the shear force in that zone. Hence the strength reduction factor φ is 0.90 for flexure failure and 0.75 for shear failure. As shear failure is uncontrolled, the factor of safety is considered 25% and so φ is more conservative in shear reinforcement design. So, shear failure is more dangerous.

Q-3. Why the critical section for shear is considered at a distance 'd' from the face of the support? [*CE-2013*]
Ans: The maximum shear force generally occurs immediately adjacent to a support. The first diagonal crack does not occur directly at the support, but at some distance from the support, of the order of the effective depth 'd' of the beam. This is so because at and adjacent to the supports the additional local stresses caused by reactions counteract crack formation. For this reason, maximum critical shear is considered at a distance 'd' from the face of the support.

Q-4. What are the types of shear reinforcement in beams may be used?
Ans: Different types of shear reinforcement may be used to resist the diagonal cracking of beam:
(i) Stirrups, which can be placed either perpendicular to the longitudinal reinforcement or inclined usually in 45° angle and welded to the main longitudinal reinforcement.
(ii) Bent bars, which are part of the longitudinal reinforcement at an angle of 45°.
(iii) Combinations of stirrups and bent bars.
(iv) Welded wire fabric with wires perpendicular to the axis of the member.
(v) Spirals, circular ties, or hoops in circular section as columns.

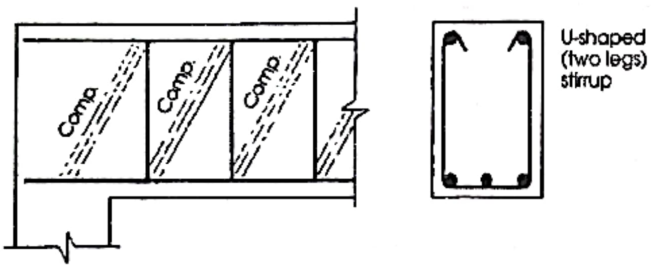


Fig: Stirrups – vertical

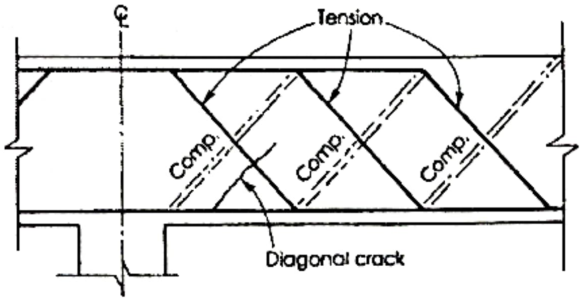
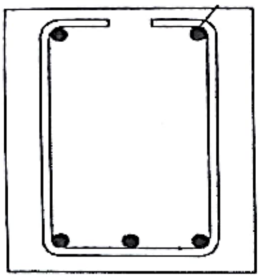


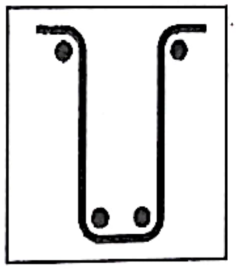
Fig: Bent bars

Q-5. Discuss the types of stirrup or web reinforcement in beams with appropriate neat sketch.

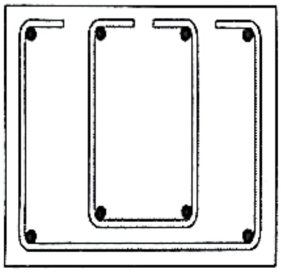
Ans: Various types of stirrup are described below:

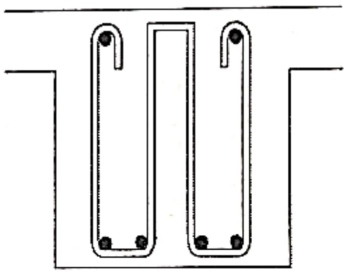
(i) Single vertical stirrup



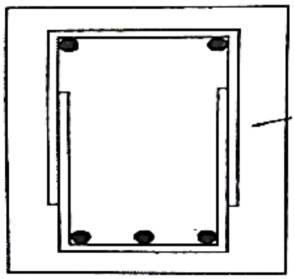


(ii) Multiple stirrup





(iii) Lap spliced stirrup



Q-6. What are the purposes of shear reinforcement or stirrups in beams?

Ans: Purposes of shear reinforcement in beams:

(i) Shear reinforcement increases the member strengths against diagonal tension failure by direct transfer of shear force and improvement of aggregate interlock.

(ii) It helps to maintain the intensity of compression zone.

(iii) It improves the concrete member ductility.

(iv) It also holds the longitudinal reinforcement on its place of proper position.

(v) It increases the stability of the structural elements and also prevents the buckling failure of column which acts as tie bar.

Q-7. Why slabs are not designed for shear reinforcement?
Ans: Shear reinforcement is not considered in the slab design for the reasons:-
(i) Depth of slab is small and span therefore fairly slender so bending and deflection will nearly always govern the design.
(ii) Most of the slabs are subjected to uniformly distributed loads. Because of this, the developed shear stresses are small.

4.2. Basic Concepts:

1. Concrete aggregate factor, λ:

Aggregate type	Aggregate factor, λ
Normal-weight concrete	1.0
Sand, CA light-weight concrete	0.85
All light-weight concrete	0.75

2. Maximum Spacing of Stirrup (S_{max}):

(a) If $\varphi V_s < \varphi 4\sqrt{f_c'}\, b_w d$
S_{max} is the least of-
i) $S = \frac{d}{2}$
ii) $S = 24\ in.$
iii) $S = \frac{A_v f_{yt}}{0.75\sqrt{f_c'}\, b_w d} \leq \frac{A_v f_{yt}}{50 b_w}$

(b) If $\varphi V_s > \varphi 4\sqrt{f_c'}\, b_w d$ & $\varphi V_s < \varphi 8\sqrt{f_c'}\, b_w d$
S_{max} is the least of- *[half of (a)]*
i) $S = \frac{d}{4}$
ii) $S = 12\ in.$
iii) $S = \frac{A_v f_{yt}}{0.75\sqrt{f_c'}\, b_w d} \leq \frac{A_v f_{yt}}{50 b_w}$

4.3. Solution Techniques:

Design of shear (web) reinforcement:

WSD

Step-1. Load diagram & SFD

Step-2. Shear and distance calculation:

Critical shear, $V = R - wd$

Shear capacity of concrete,

$$V_c = 1.1\sqrt{f_c'}\, b_w d$$

Shear acts at a distance $x = \frac{L}{2}\left(\frac{R-\varphi V_c}{R}\right)$; [from similar triangle]
Shear reinforcement is carried at a total distance of $(x + d)$.

Step-3. Maximum spacing:

S_{max} is the least of-

i) $S = \frac{d}{2}$

ii) $S = 24\ in.$

iii) $S = \frac{A_v}{0.0015\, b_w}$

Step-4. Determining shear & location for S_{max},

$$S = \frac{A_v\, f_v d}{V - V_c}$$

Step-5. Average shear of V (critical) & V (S_{max}), distance and spacing, S

Step-6. Spacing, S for critical shear, V

Step-7. Reinforcement detailing:
Step-8. Stirrup distribution:

USD

Step-1. Load diagram & SFD
Step-2. Shear and distance calculation:
Critical shear, $V_u = R - wd$
Shear capacity of concrete,

$$\varphi V_c = \varphi 2\lambda\sqrt{f_c'}\, b_w d$$

If $V_u < \frac{\varphi V_c}{2}$; no shear reinforcement is needed.

Shear acts at a distance $= \frac{L}{2}\left(\frac{R-\varphi V_c}{R}\right)$; [from similar triangle]
Shear reinforcement must be provided where shear exceeds $\frac{\varphi V_c}{2}$.

Step-3. Check for permissible shear:
Shear resisted by shear reinforcement,

$$\varphi V_s = V_u - \varphi V_c$$

Maximum permissible value $\varphi 4\sqrt{f_c'}\, b_w d$ & $\varphi 8\sqrt{f_c'}\, b_w d$, applying condition for S_{max}
Step-4. Maximum spacing:
S_{max} is the least of-
i) $S = \frac{d}{2}$
ii) $S = 24\ in.$
iii) $S = \frac{A_v\, f_{yt}}{0.75\sqrt{f_c'}\, b_w d} \leq \frac{A_v\, f_{yt}}{50 b_w}$
Step-5. Determining shear & location for S_{max},

$$S = \frac{\varphi A_v\, f_{yt} d}{V_u - \varphi V_c}$$

Step-6. Average shear of V_u (critical) & V_u (S_{max}) and spacing, S.
Step-7. Spacing, S for critical shear, V_u
Step-8. Reinforcement detailing:
Step-9. Stirrup distribution:

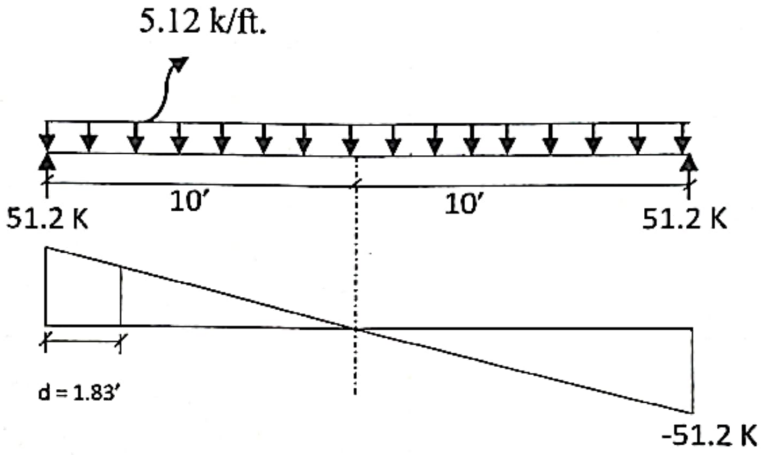
4.4. Problem Solution:

4.4.1 Design of shear (web) reinforcement - WSD

Pb-1. A simply supported rectangular beam of b = 16 in. and d = 22 in. carries a total working load of 5.12 k/ft. on a 20 ft. clear span. Design shear reinforcement for the beam by WSD method. Use $f_c' = 3\ ksi\ and\ f_y = 50\ ksi$. [*CE-2019, 2015(sim), 2012*]

Sol^n:

1. Load diagram & SFD:



2. Shear and distance calculation:

Critical shear acts at d = 22" = 1.83 ft from the face of the support,

$$V = R - wd = 51.2 - 5.12 \times 1.83 = 41.8\ kips$$

∴ Shear capacity of concrete,

$$V_c = 1.1\sqrt{f_c'}\, b_w d = 1.1\sqrt{3000} \times 16 \times 22 = 21.2\ kips$$

It acts at a distance $10\left(\frac{51.2-21.2}{51.2}\right) = 5.86\ ft$ from the face of the support.

[from similar triangle]

Shear reinforcement is carried at a total distance of $(5.86 + 1.83) = 7.69\ ft$

3. Maximum spacing:

S_{max} is the least of-

$i)\ S = \frac{d}{2} = \frac{22}{2} = 11\ in.$

$ii)\ S = 24\ in.$

$iii)\ S = \frac{A_v}{0.0015\ b_w} = \frac{2 \times 0.11}{0.0015 \times 16} = 9.2\ in.$

$\therefore S_{max} = 9.2\ in.$

4. Determining shear & location for S_{max}, (Taking #3 stirrups for trial)

$$S = \frac{A_v\ f_v d}{V - V_c}$$

$$\Rightarrow V - V_c = \frac{A_v\ f_v d}{S}$$

$$\Rightarrow V = \frac{A_v\ f_v d}{S} + V_c$$

$$\Rightarrow V = \frac{(2 * 0.11) \times 20 \times 22}{9.2} + 21.2\ ;\quad [f_v = 0.4 f_y = 20\ ksi]$$

$\therefore V = 31.72\ kips$

It acts at a distance $10\left(\frac{51.2-31.72}{51.2}\right) = 3.80\ ft$ from the face of the support.

5. Average shear of 41.8 kips & 31.96 kips is 36.88 kips.

It acts at a distance $10\left(\frac{51.2-36.88}{51.2}\right) = 2.80\ ft$ from the face of the support.

$\therefore$ Spacing,

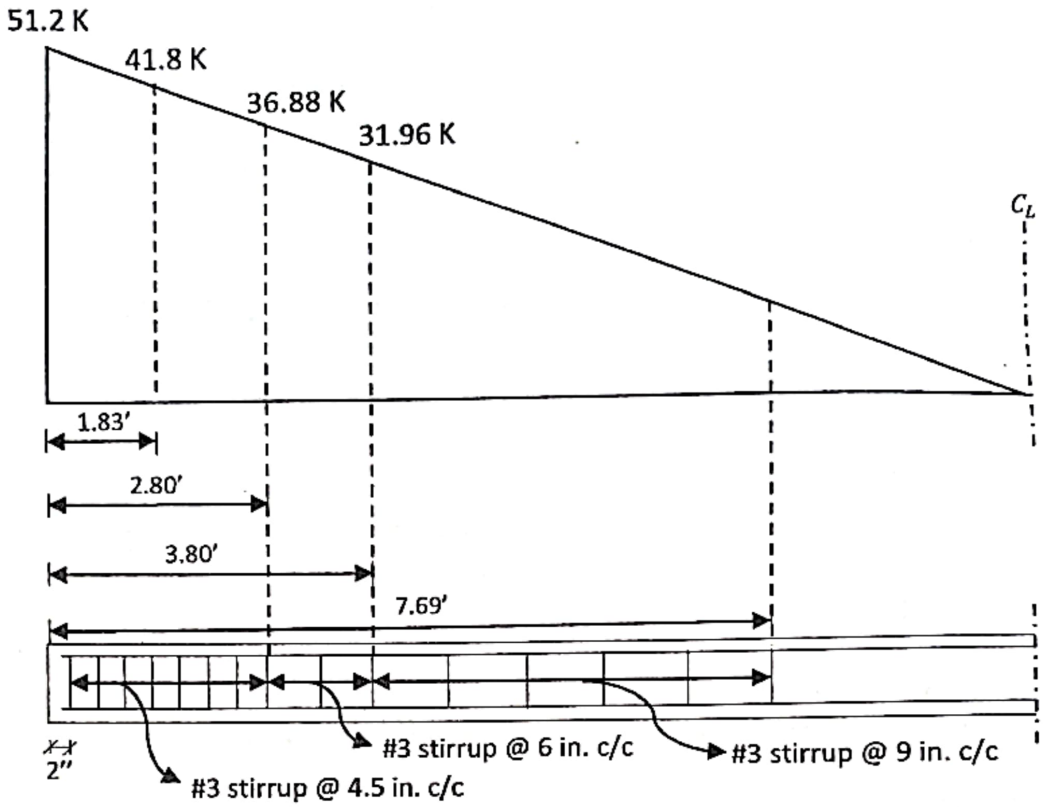
$$S = \frac{A_v\ f_v d}{V - V_c} = \frac{(2 * 0.11) \times 20 \times 22}{36.88 - 21.2} = 6.17\ in. \approx 6\ in.$$

6. Spacing for critical shear V is,

$$S_1 = \frac{A_v\ f_v d}{V - V_c} = \frac{(2 * 0.11) \times 20 \times 22}{41.8 - 21.2} = 4.69\ in. \approx 4.5\ in.$$

The first stirrup is placed at distance $\frac{S_1}{2} = \frac{4.5}{2} = 2.25 \approx 2\ in.$ from the face of the support.

7. Reinforcement detailing:



8. Stirrup distribution:

i) 1 First stirrup @ 2 in. = 2 in.

ii) 7 stirrup @ 4.5 in. = 31.5 in.

iii) 2 stirrup @ 6 in. = 12 in.

iv) 5 stirrup @ 9 in. = 45 in.

Total = 90.5 in. = 7.54 ft (≈7.69 ft) (Answer)

Easy rough calculation:

For (ii), Stirrup number $= \frac{distance}{spacing} = \frac{(2.80\times12)-2\ in.}{4.5\ in.} = 7.02 \approx 7\ nos.$ & $length = 7 \times 4.5 = 31.5\ in.$

For (iii), Stirrup number $= \frac{distance}{spacing} = \frac{(3.80-2.80)\times12\ in.}{6\ in.} = 2\ nos.$ & $length = 2 \times 6 = 12\ in.$

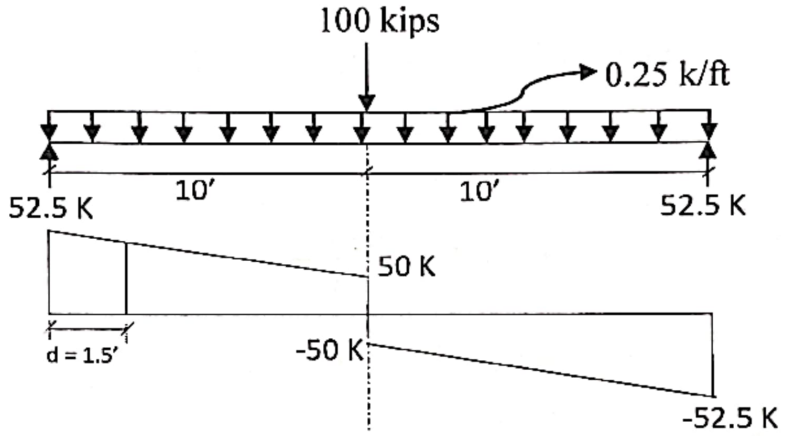
For (iv), Stirrup number $= \frac{distance}{spacing} = \frac{(7.69-3.80)\times12\ in.}{9\ in.} = 5.19 \approx 5\ nos.$ & $length = 5 \times 9 = 45\ in.$

Pb-2. A simply supported beam 12 in. wide by 18 in. effective depth, having clear span of 20 ft. between supports carries a concentrated load of 100 kips at its midpoint in addition to its own weight. Design shear reinforcement for the beam by WSD method. Use $f_c' = 4000\ psi\ and\ f_y = 60{,}000\ psi$. [***CE-2017***]

Sol^n:

1. Load diagram & SFD:

Self-weight $\frac{12 \times 20}{144} \times 150 = 250\ lb/ft = 0.25\ k/ft$; [Total ht, h = d + cc = 18 + 2 = 20 in.]



2. Shear and distance calculation:

Critical shear acts at d = 18″ = 1.5 ft from the face of the support,

$$V = R - wd = 52.5 - 0.25 \times 1.5 = 52.13\ kips$$

∴ Shear capacity of concrete,

$$V_c = 1.1\sqrt{f_c'}\, b_w d = 1.1\sqrt{4000} \times 10 \times 18 = 12.52\ kips$$

It acts at a distance $10\left(\frac{52.5 - 12.52}{52.5 - 50}\right) = 12.3\ ft$ from the face of the support.

[from similar triangle]

Shear reinforcement is carried at a total distance of $(12.3 + 1.5) = 13.8\ ft$

Here, V_c exceeds the shear of 50 kips & distance of C_L = 10 ft,

So, Shear reinforcement have to provide to centerline along all the span length.

3. Maximum spacing:

S_{max} is the least of-

$i)\ S = \frac{d}{2} = \frac{18}{2} = 9\ in.$

$ii)\ S = 24\ in.$

$iii)\ S = \frac{A_v}{0.0015\ b_w} = \frac{2 \times 0.11}{0.0015 \times 10} = 14.67\ in.$

$\therefore S_{max} = 9\ in.$

4. Determining shear & location for S_{max}, (Taking #3 stirrups for trial)

$$S = \frac{A_v\ f_v d}{V - V_c}$$

$$\Rightarrow V - V_c = \frac{A_v\ f_v d}{S}$$

$$\Rightarrow V = \frac{A_v\ f_v d}{S} + V_c$$

$$\Rightarrow V = \frac{(2 * 0.11) \times 24 \times 18}{9} + 12.52\ ;\quad [f_v = 0.4\ f_y = 24\ ksi]$$

$\therefore V = 23.08\ kips$

It acts at a distance $10\left(\frac{52.5-23.08}{52.5-50}\right) = 117.68\ ft$ from the face of the support.

5. Average shear of 52.13 kips & 23.08 kips = 37.61 kips.

It acts at a distance $10\left(\frac{52.5-37.61}{52.5-50}\right) = 59.56\ ft$ from the face of the support.

$\therefore$ Spacing,

$$S = \frac{A_v\ f_v d}{V - V_c} = \frac{(2 * 0.11) \times 24 \times 18}{37.61 - 12.52} = 3.79\ in. \approx 3.5\ in.$$

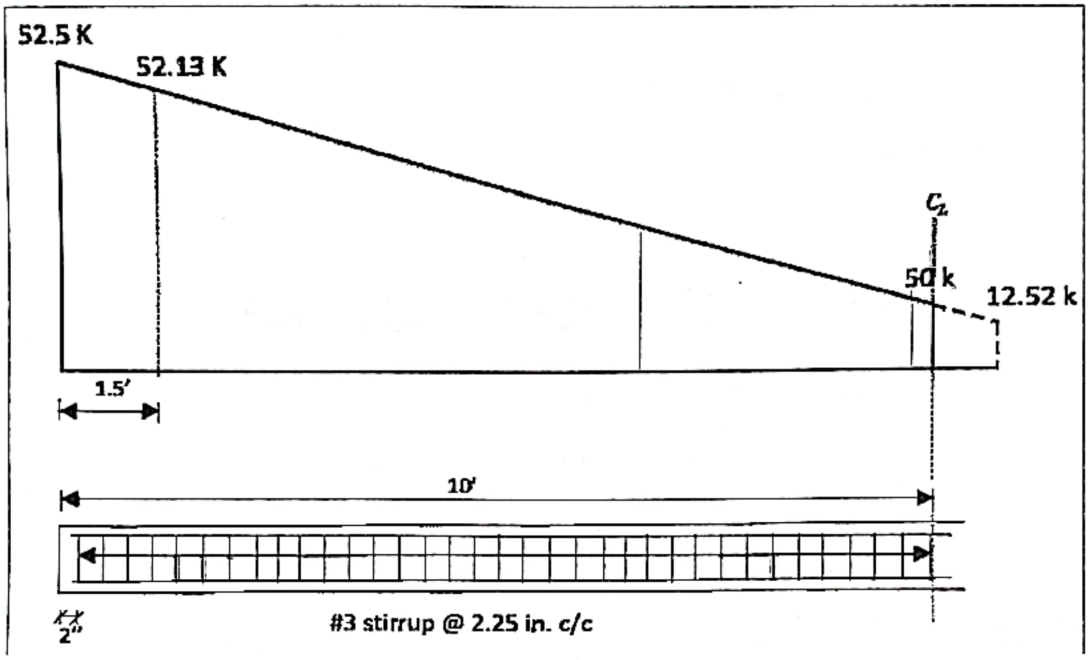
6. Spacing for critical shear V is,

$$S_1 = \frac{A_v\ f_v d}{V - V_c} = \frac{(2 * 0.11) \times 24 \times 18}{52.13 - 12.52} = 2.4\ in. \approx 2.25\ in.$$

The first stirrup is placed at distance $\frac{S_1}{2} = \frac{2.25}{2} = 1.13 \approx 2\ in.$ from the face of the support.

Every location of shear exceeds the centerline distance. So, the spacing of stirrup should be chosen of the spacing (S_1) that is calculated for critical shear.

7. Reinforcement detailing:



8. Stirrup distribution:

i) 1 first stirrup @ 2 in. = 2 in.

ii) 52 stirrup @ 2.25 in = 117 in.

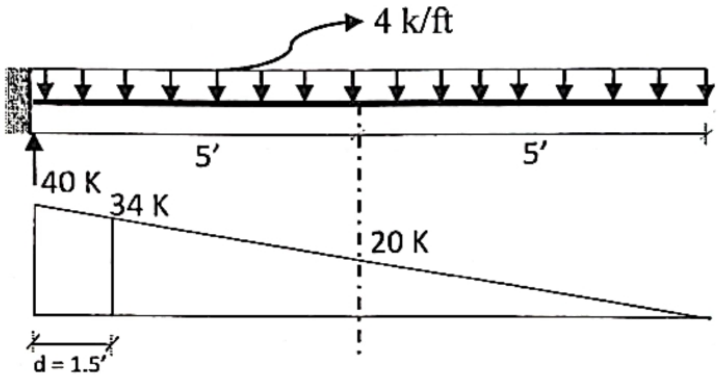
Total = 119 in. = 9.92 ft. (≈10 ft)

Pb-3. A cantilever beam having b = 10 in. and h = 20 in. carries a working live load of 4 k/ft on a 10 ft span. Design shear reinforcement for the beam by WSD method. Use $f_c' = 3000\ psi$ *and* $f_y = 60{,}000\ psi$. Show reinforcement detail in longitudinal and cross-section. [*CE-2014*]

Sol^n:

Effective depth, d = h – cc = 20 – 2 = 18 in.

1. Load diagram & SFD



2. Shear and distance calculation:

Critical shear acts at d = 18" = 1.5 ft from the face of the support,

$$V = R - wd = 40 - 4 \times 1.5 = 34\ kips$$

∴ Shear capacity of concrete,

$$V_c = 1.1\sqrt{f_c'}\, b_w d = 1.1\sqrt{3000} \times 10 \times 18 = 10.84\ kips$$

It acts at a distance $10\left(\frac{40-10.84}{40}\right) = 7.29\ ft$ from the face of the support.

[from similar triangle]

Shear reinforcement is carried at a total distance of $(7.29 + 1.5) = 8.79\ ft$

3. Maximum spacing:

S_{max} is the least of-

i) $S = \frac{d}{2} = \frac{18}{2} = 9\ in.$

$ii)\ S = 24\ in.$

$iii)\ S = \dfrac{A_v}{0.0015\ b_w} = \dfrac{2 \times 0.11}{0.0015 \times 10} = 14.67\ in.$

$\therefore S_{max} = 9\ in.$

4. Determining shear & location for S_{max}, (Taking #3 stirrups for trial)

$$S = \frac{A_v\ f_v d}{V - V_c}$$

$$\Rightarrow V - V_c = \frac{A_v\ f_v d}{S}$$

$$\Rightarrow V = \frac{A_v\ f_v d}{S} + V_c$$

$$\Rightarrow V = \frac{(2 * 0.11) \times 24 \times 18}{9} + 10.84\ ; \quad [f_v = 0.4\ f_y = 20\ ksi]$$

$\therefore V = 21.4\ kips$

It acts at a distance $10\left(\frac{40-21.4}{40}\right) = 4.65\ ft$ from the face of the support.

5. Average shear of 34 kips & 21.4 kips = 27.7 kips.

It acts at a distance $10\left(\frac{40-27.7}{40}\right) = 3.1\ ft$ from the face of the support.

∴ Spacing,

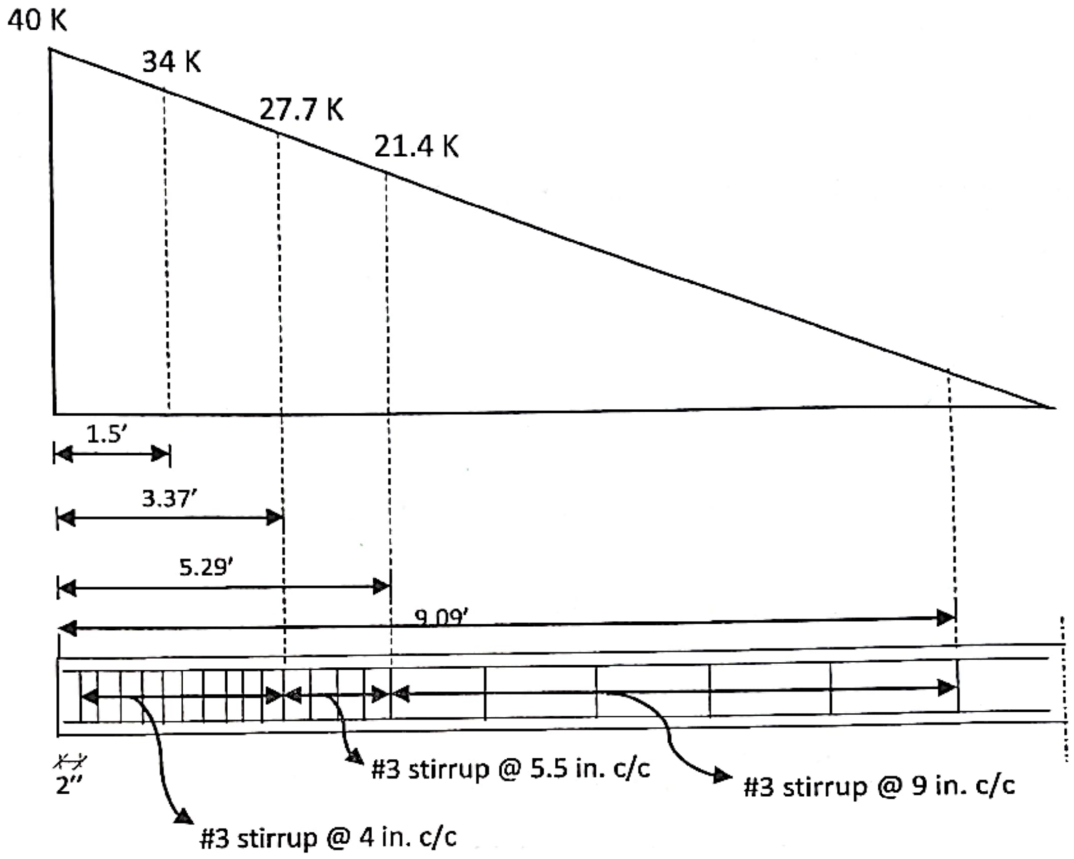
$$S = \frac{A_v\ f_v d}{V - V_c} = \frac{(2 * 0.11) \times 24 \times 18}{27.7 - 10.84} = 5.64\ in. \approx 5.5\ in.$$

6. Spacing for critical shear V is,

$$S_1 = \frac{A_v\ f_v d}{V - V_c} = \frac{(2 * 0.11) \times 24 \times 18}{34 - 10.84} = 4.1\ in. \approx 4\ in.$$

The first stirrup is placed at distance $\frac{S_1}{2} = \frac{4}{2} = 2\ in.$ from the face of the support.

7. Reinforcement detailing:



8. Stirrup distribution:

i) 1 First stirrup @ 2 in. = 2 in.

ii) 10 stirrup @ 4 in. = 40 in.

iii) 4 stirrup @ 5.5 in. = 22 in.

iv) 5 stirrup @ 9 in. = 45 in.

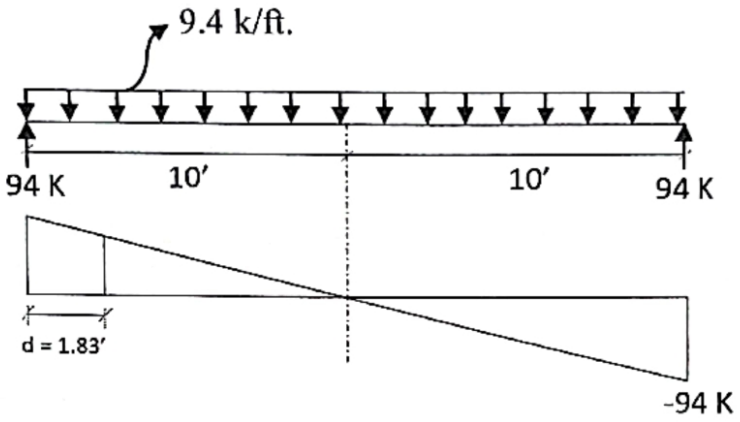
Total = 109 in. = 9.083 ft (≈9.09 ft) (Answer)

4.4.2 Design of shear (web) reinforcement - *USD*

Pb-4. A simply supported rectangular beam of b = 16 in. and d = 22 in. carries a total factored load of 9.4 k/ft. on a 20 ft. clear span. Design shear reinforcement for the beam by USD method. Use $f_c' = 4000\ psi\ and\ f_y = 60{,}000\ psi$. ***[BECM-2018(sim)]***

Sol^n:

1. Load diagram & SFD:



2. Shear and distance calculation:

Critical shear acts at d = 22″ = 1.83 f.t from the face of the support,

$V_u = R - wd = 94 - 9.4 \times 1.83 = 76.8\ kips$

∴ Shear capacity of concrete,

$\varphi V_c = \varphi 2\lambda\sqrt{f_c'}\, b_w d = 0.75 \times 2 \times 1 \times \sqrt{4000} \times 16 \times 22 = 33.4\ kips$

It acts at a distance $10\left(\frac{94-33.4}{94}\right) = 6.45\ ft$ from the face of the support.

[from similar triangle]

Here, $\frac{\varphi V_c}{2} = \frac{33.4}{2} = 16.7\ kips < 76.8\ kips$

∴ $V_u \not< \frac{\varphi V_c}{2}$; shear reinforcement is needed.

According to ACI code,

Shear reinforcement must be provided where shear exceeds $\frac{\varphi V_c}{2} = 16.7\ kips$.

It acts at a distance $10\left(\frac{94-16.7}{94}\right) = 8.22\ ft$ from the face of the support.

3. Check for permissible shear:

Shear resisted by shear reinforcement,

$\varphi V_s = V_u - \varphi V_c = 76.8 - 33.4 = 43.4\ kips$

Maximum permissible value $= \varphi 4\sqrt{f_c'}\, b_w d = 0.75 \times 4 \times \sqrt{4000} \times 16 \times 22 = 68.8\ kips$

So, $\varphi V_s < \varphi 4\sqrt{f_c'}\, b_w d$

4. Maximum spacing:

S_{max} is the least of-

$i)\ S = \frac{d}{2} = \frac{22}{2} = 11\ in.$

$ii)\ S = 24\ in.$

$iii)\ S = \frac{A_v f_{yt}}{0.75\sqrt{f_c'}\, b_w} \leq \frac{A_v f_{yt}}{50 b_w} = \frac{(2 * 0.11) \times 60{,}000}{0.75\sqrt{4000} \times 16} \leq \frac{(2 * 0.11) \times 60{,}000}{50 \times 16}$

$= 17.4\ in. \leq 16.5\ in.$

$\therefore S_{max} = 11\ in.$

5. Determining shear & location for S_{max},

$S = \frac{\varphi A_v f_{yt} d}{V_u - \varphi V_c}$

$\Rightarrow V_u - \varphi V_c = \frac{\varphi A_v f_{yt} d}{S}$

$\Rightarrow V_u = \frac{\varphi A_v f_{yt} d}{S} + \varphi V_c$

$\Rightarrow V_u = \frac{0.75 \times (2 * 0.11) \times 60 \times 22}{11} + 33.4$

$\therefore V_u = 53.2\ kips$

It acts at a distance $10\left(\frac{94-53.2}{94}\right) = 4.35\ ft$ from the face of the support.

6. Average shear of 76.8 kips & 53.2 kips is 65 kips

It acts at a distance $10\left(\frac{94-65}{94}\right) = 3.1\ ft$ from the face of the support.

$\therefore$ Spacing,

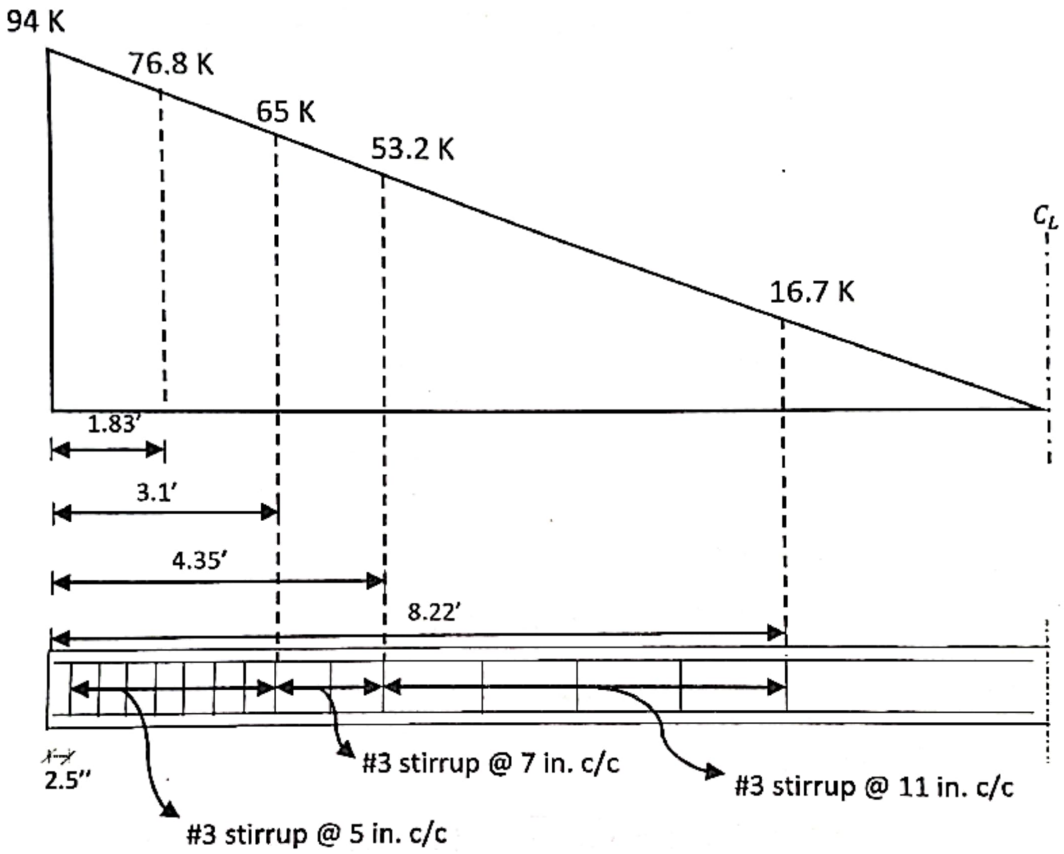
$$S = \frac{\varphi A_v f_{yt} d}{V_u - \varphi V_c} = \frac{0.75 \times (2 * 0.11) \times 60 \times 22}{65 - 33.4} = 6.89\ in. \approx 7\ in.$$

7. Spacing for critical shear V is,

$$S_1 = \frac{\varphi A_v f_{yt} d}{V_u - \varphi V_c} = \frac{0.75 \times (2 * 0.11) \times 60 \times 22}{76.8 - 33.4} = 5.02\ in. \approx 5\ in.$$

The first stirrup is placed at distance $\frac{S_1}{2} = \frac{5}{2} = 2.5\ in.$ from the face of the support.

8. Reinforcement detailing:



9. Stirrup distribution:

i) 1 first stirrup @ 2.5 in. = 2.5 in.

ii) 7 stirrup @ 5 in. = 35 in.

iii) 2 stirrup @ 7 in. = 14 in.

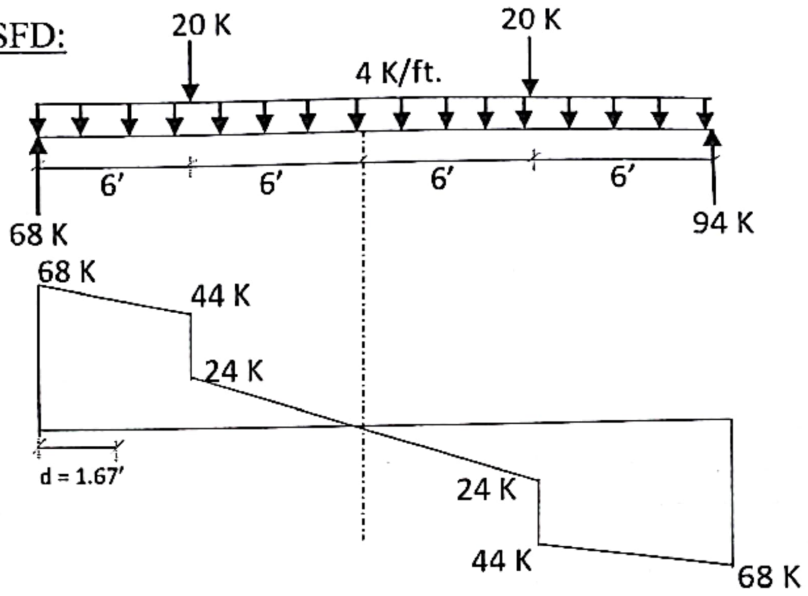
iv) 4 stirrup @ 11 in. = 44 in.

Total = 95.5 in. = 7.96 ft (≈8.22 ft)

Pb-5. A simply supported rectangular beam of b = 10 in. and d = 20 in. shows the load diagram as follows. Design shear reinforcement for the beam by USD method. Use $f_c' = 3000\ psi\ and\ f_y = 60{,}000\ psi.$

Sol^n:

1. Load diagram & SFD:



2. Shear and distance calculation:

Critical shear acts at d = 20" = 1.67 ft. from the face of the support,

$V_u = R - wd = 68 - 4 \times 1.67 = 61.32\ kips$

∴ Shear capacity of concrete,

$\varphi V_c = \varphi 2\lambda\sqrt{f_c'}\, b_w d = 0.75 \times 2 \times 1 \times \sqrt{3000} \times 10 \times 20 = 16.43\ kips$

It acts at a distance $6 + 6\left(\frac{24-16.43}{24}\right) = 7.89\ ft$ from the face of the support.

[from similar triangle]

Here, $\frac{\varphi V_c}{2} = \frac{16.43}{2} = 8.22\ kips < 61.32\ kips$

∴ $V_u \not< \frac{\varphi V_c}{2}$; shear reinforcement is needed.

According to ACI code,

Shear reinforcement must be provided where shear exceeds $\frac{\varphi V_c}{2} = 8.22\ kips.$

It acts at a distance $6 + 6\left(\frac{24-8.22}{24}\right) = 9.95\ ft$ from the face of the support.

3. Check for permissible shear:

Shear resisted by shear reinforcement,

$\varphi V_s = V_u - \varphi V_c = 61.32 - 16.43 = 44.89\ kips$

Maximum permissible value $= \varphi 4\sqrt{f_c'}\, b_w d = 0.75 \times 4 \times \sqrt{3000} \times 10 \times 20 = 32.86\ kips$

So, $\varphi V_s \not< \varphi 4\sqrt{f_c'}\, b_w d$

And, $\varphi 8\sqrt{f_c'}\, b_w d = 0.75 \times 8 \times \sqrt{3000} \times 10 \times 20 = 65.73\ kips$

Here, $\varphi V_s < \varphi 8\sqrt{f_c'}\, b_w d$

4. Maximum spacing:

S_{max} is the least of-

$i)\ S = \frac{d}{4} = \frac{20}{4} = 5\ in.$

$ii)\ S = 12\ in.$

$iii)\ S = \frac{A_v f_{yt}}{0.75\sqrt{f_c'}\, b_w} \leq \frac{A_v f_{yt}}{50 b_w} = \frac{(2 * 0.11) \times 60{,}000}{0.75\sqrt{4000} \times 16} \leq \frac{(2 * 0.11) \times 60{,}000}{50 \times 16}$

$= 17.4\ in. \leq 16.5\ in.$

$\therefore S_{max} = 5\ in.$

5. Determining shear & location for S_{max},

$S = \frac{\varphi A_v f_{yt} d}{V_u - \varphi V_c}$

$\Rightarrow V_u - \varphi V_c = \frac{\varphi A_v f_{yt} d}{S}$

$\Rightarrow V_u = \frac{\varphi A_v f_{yt} d}{S} + \varphi V_c$

$\Rightarrow V_u = \frac{0.75 \times (2 * 0.11) \times 60 \times 20}{5} + 16.43$

$\therefore V_u = 54.45\ kips$

It acts at a distance $6\left(\frac{68-54.45}{68-44}\right) = 3.39\ ft$ from the face of the support.

6. Average shear of 61.32 kips & 54.45 kips is 57.89 kips

It acts at a distance $6\left(\frac{68-57.89}{68-44}\right) = 2.53\ ft$ from the face of the support.

∴ Spacing,

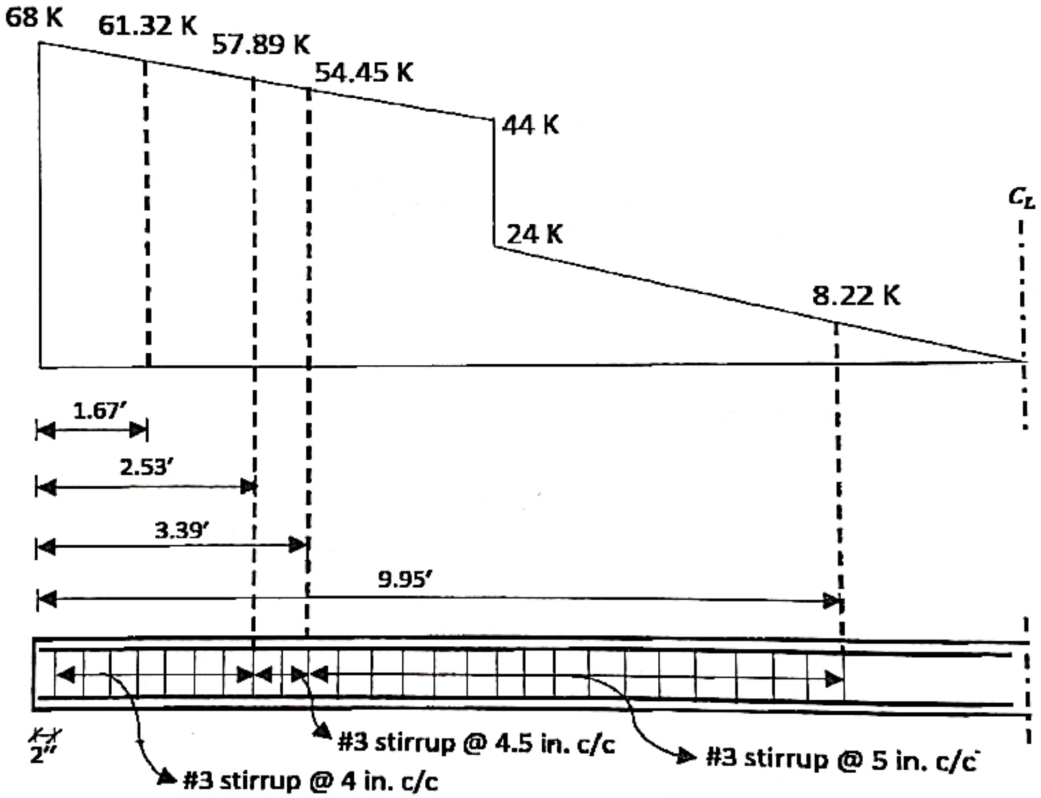
$$S = \frac{\varphi A_v f_{yt} d}{V_u - \varphi V_c} = \frac{0.75 \times (2 * 0.11) \times 60 \times 20}{57.89 - 16.43} = 4.76\ in. \approx 4.5\ in.$$

7. Spacing for critical shear V is,

$$S_1 = \frac{\varphi A_v f_{yt} d}{V_u - \varphi V_c} = \frac{0.75 \times (2 * 0.11) \times 60 \times 20}{61.32 - 33.4} = 4.4\ in. \approx 4\ in.$$

The first stirrup is placed at distance $\frac{S_1}{2} = \frac{4}{2} = 2\ in.$ from the face of the support.

8. Reinforcement detailing:



9. Stirrup pattern:

i) 1 first stirrup @ 2 in. = 2 in.

ii) 7 stirrup @ 4 in. = 28 in.

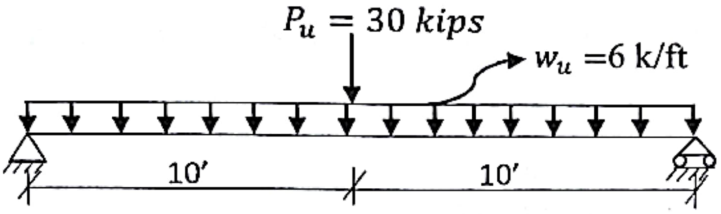
iii) 2 stirrup @ 4.5 in. = 9 in.

iv) 16 stirrup @ 5 in. = 80 in.

Total = 119 in. = 9.92 ft. (≈9.95 ft)

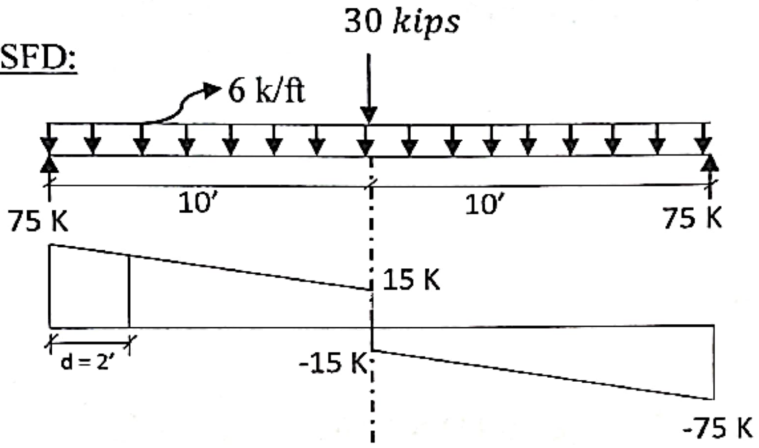
Pb-6. Design shear reinforcement for the following simply supported beam of b = 12 in. and d = 24 in. Use $f_c' = 3500\ psi\ and\ f_y = 60{,}000\ psi$. Follow USD method.

[*CE-2018; BECM-2019(sim)*]



Sol^n:

1. Load diagram & SFD:



2. Shear and distance calculation:

Critical shear acts at d = 24″ = 2 ft. from the face of the support,

$V_u = R - wd = 75 - 6 \times 2 = 63\ kips$

∴ Shear capacity of concrete,

$\varphi V_c = \varphi 2\lambda\sqrt{f_c'}\, b_w d = 0.75 \times 2 \times 1 \times \sqrt{3500} \times 12 \times 24 = 25.56\ kips$

It acts at a distance $10\left(\frac{75-25.56}{75-15}\right) = 8.24\ ft$ from the face of the support. [from similar triangle]

Here, $\frac{\varphi V_c}{2} = \frac{25.56}{2} = 12.78\ kips < 63\ kips$

∴ $V_u \not< \frac{\varphi V_c}{2}$; shear reinforcement is needed.

According to ACI code,

Shear reinforcement must be provided where shear exceeds $\frac{\varphi V_c}{2} = 12.78\ kips$.

It acts at a distance $10\left(\frac{75-12.78}{75-15}\right) = 10.37\ ft$ from the face of the support.

Here, $\frac{\varphi V_c}{2}$ cross the shear of 15 kips & distance of C_L = 10 ft,

So, Shear reinforcement have to provide to centerline along all the span length.

3. Check for permissible shear:

Shear resisted by shear reinforcement,
$\varphi V_s = V_u - \varphi V_c = 63 - 25.56 = 37.44\ kips$
Maximum permissible value = $\varphi 4\sqrt{f_c'}\, b_w d = 0.75 \times 4 \times \sqrt{3500} \times 12 \times 24 = 51.11\ kips$
So, $\varphi V_s < \varphi 4\sqrt{f_c'}\, b_w d$

4. Maximum spacing:

S_{max} is the least of-

$i)\ S = \frac{d}{2} = \frac{24}{2} = 12\ in.$

$ii)\ S = 24\ in.$

$iii)\ S = \frac{A_v f_{yt}}{0.75\sqrt{f_c'}\, b_w} \leq \frac{A_v f_{yt}}{50 b_w} = \frac{(2 * 0.11) \times 60{,}000}{0.75\sqrt{3500} \times 12} \leq \frac{(2 * 0.11) \times 60{,}000}{50 \times 12}$

$= 24.79\ in. \leq 22\ in.$

$\therefore S_{max} = 12\ in.$

5. Determining shear & location for S_{max},

$$S = \frac{\varphi A_v f_{yt} d}{V_u - \varphi V_c}$$

$$\Rightarrow V_u - \varphi V_c = \frac{\varphi A_v f_{yt} d}{S}$$

$$\Rightarrow V_u = \frac{\varphi A_v f_{yt} d}{S} + \varphi V_c$$

$$\Rightarrow V_u = \frac{0.75 \times (2 * 0.11) \times 60 \times 24}{12} + 25.56$$

$\therefore V_u = 45.36\ kips$

It acts at a distance $10\left(\frac{75-45.36}{75-15}\right) = 4.94\ ft$ from the face of the support.

6. Average shear of 63 kips & 45.36 kips is 54.18 kips

It acts at a distance $10\left(\frac{75-54.18}{75-15}\right) = 3.47\ ft$ from the face of the support.

∴ Spacing,

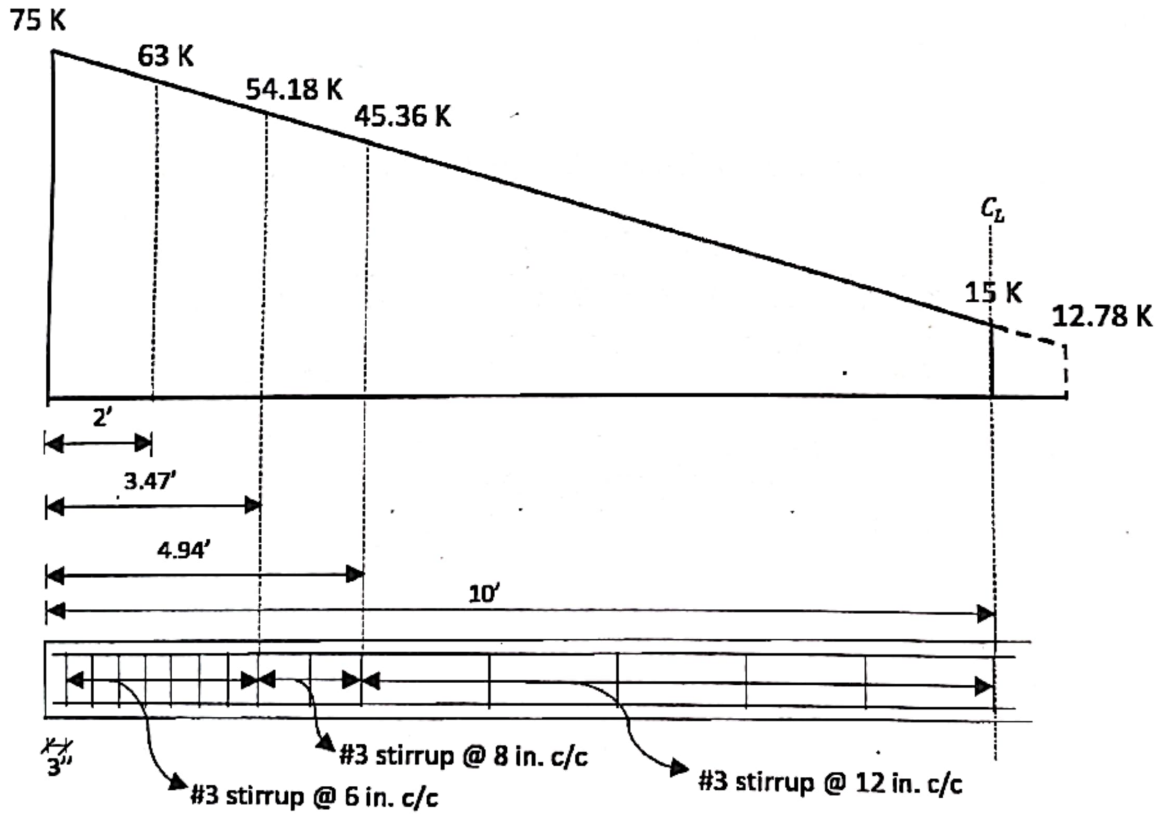
$$S = \frac{\varphi A_v f_{yt} d}{V_u - \varphi V_c} = \frac{0.75 \times (2 * 0.11) \times 60 \times 24}{54.18 - 25.56} = 8.3\ in. \approx 8\ in.$$

7. Spacing for critical shear V is,

$$S_1 = \frac{\varphi A_v f_{yt} d}{V_u - \varphi V_c} = \frac{0.75 \times (2 * 0.11) \times 60 \times 24}{63 - 25.56} = 6.35\ in. \approx 6\ in.$$

The first stirrup is placed at distance $\frac{S_1}{2} = \frac{6}{2} = 3\ in.$ from the face of the support.

8. <u>Reinforcement detailing:</u>



9. Stirrup distribution:

i) 1 first stirrup @ 3 in. = 3 in.

ii) 7 stirrup @ 6 in. = 42 in.

iii) 2 stirrup @ 8 in. = 16 in.

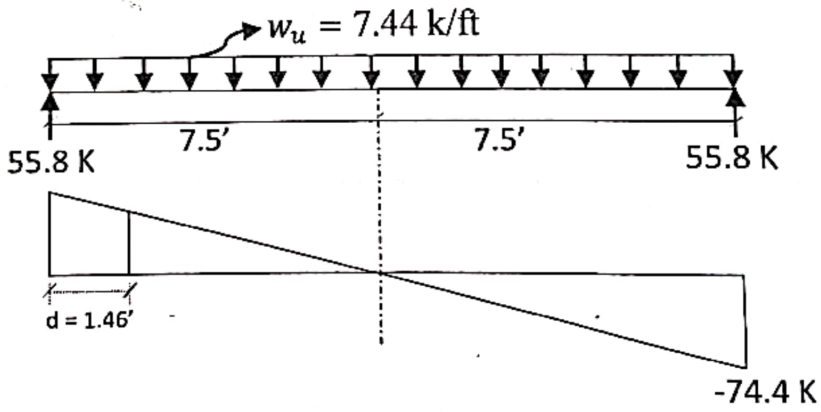
iv) 5 stirrup @ 12 in. = 60 in.

Total = 121 in. = 10.1 ft (≈10 ft)

Pb-7. A rectangular beam having b = 10 in. and d = 17.5 in. spans 15 ft face to face of simple supports. It is reinforced for flexure with three No. 9 bars that continue uninterrupted to the ends of the span. It is to carry service dead load of 1.27 k/ft (including self-weight) and service live load of 3.70 k/ft, both uniformly distributed along the span. Design the shear reinforcement for the beam using $f_c' = 4000\ psi\ and\ f_y = 60{,}000\ psi.$ [***CE-2013***]

Sol^n:

1. Load diagram & SFD: Factored uniform load, $w_u = 1.2 \times 1.27 + 1.6 \times 3.70 = 7.44\ k/ft$



2. Shear and distance calculation:

Critical shear acts at d = 17.5″ = 1.46 ft from the face of the support,

$V_u = R - wd = 55.8 - 7.44 \times 1.46 = 44.94\ kips$

∴ Shear capacity of concrete,

$$\varphi V_c = \varphi 2\lambda\sqrt{f_c'}\, b_w d = 0.75 \times 2 \times 1 \times \sqrt{4000} \times 10 \times 17.5 = 16.6\ kips$$

It acts at a distance $7.5\left(\frac{55.8-16.6}{55.8}\right) = 5.27\ ft$ from the face of the support.

[from similar triangle]

Here, $\frac{\varphi V_c}{2} = \frac{16.6}{2} = 8.3\ kips < 44.94\ kips$

$\therefore V_u > \frac{\varphi V_c}{2}$; shear reinforcement is needed.

According to ACI code,
Shear reinforcement must be provided where shear exceeds $\frac{\varphi V_c}{2} = 8.3\ kips.$

It acts at a distance $7.5\left(\frac{55.8-8.3}{55.8}\right) = 6.38\ ft$ from the face of the support.

3. Check for permissible shear:

Shear resisted by shear reinforcement,
$\varphi V_s = V_u - \varphi V_c = 44.94 - 16.6 = 28.34\ kips$

Maximum permissible value $= \varphi 4\sqrt{f_c'}\, b_w d = 0.75 \times 4 \times \sqrt{4000} \times 10 \times 17.5 = 33.2\ kips$
So, $\varphi V_s < \varphi 4\sqrt{f_c'}\, b_w d$

4. Maximum spacing:
S_{max} is the least of-

$i)\ S = \frac{d}{2} = \frac{17.5}{2} = 8.75\ in.$

$ii)\ S = 24\ in.$

$iii)\ S = \frac{A_v f_{yt}}{0.75\sqrt{f_c'}\, b_w} \leq \frac{A_v f_{yt}}{50 b_w} = \frac{(2 * 0.11) \times 60{,}000}{0.75\sqrt{4000} \times 10} \leq \frac{(2 * 0.11) \times 60{,}000}{50 \times 10}$

$= 27.83\ in. \leq 26.4\ in.$

$\therefore S_{max} = 8.75\ in.$

5. Determining shear & location for S_{max},

$S = \frac{\varphi A_v f_{yt} d}{V_u - \varphi V_c}$

$\Rightarrow V_u - \varphi V_c = \frac{\varphi A_v f_{yt} d}{S}$

$$\Rightarrow V_u = \frac{\varphi A_v f_{yt} d}{S} + \varphi V_c$$

$$\Rightarrow V_u = \frac{0.75 \times (2 * 0.11) \times 60 \times 17.5}{8.75} + 16.6$$

$$\therefore V_u = 36.4\ kips$$

It acts at a distance $7.5\left(\frac{55.8 - 36.4}{55.8}\right) = 2.61\ ft$ from the face of the support.

6. Average shear of 44.94 kips & 36.4 kips = 40.67 kips

It acts at a distance $7.5\left(\frac{55.8 - 40.7}{55.8}\right) = 2.03\ ft$ from the face of the support.

∴ Spacing,

$$S = \frac{\varphi A_v f_{yt} d}{V_u - \varphi V_c} = \frac{0.75 \times (2 * 0.11) \times 60 \times 17.5}{40.67 - 16.6} = 7.2\ in. \approx 7\ in.$$

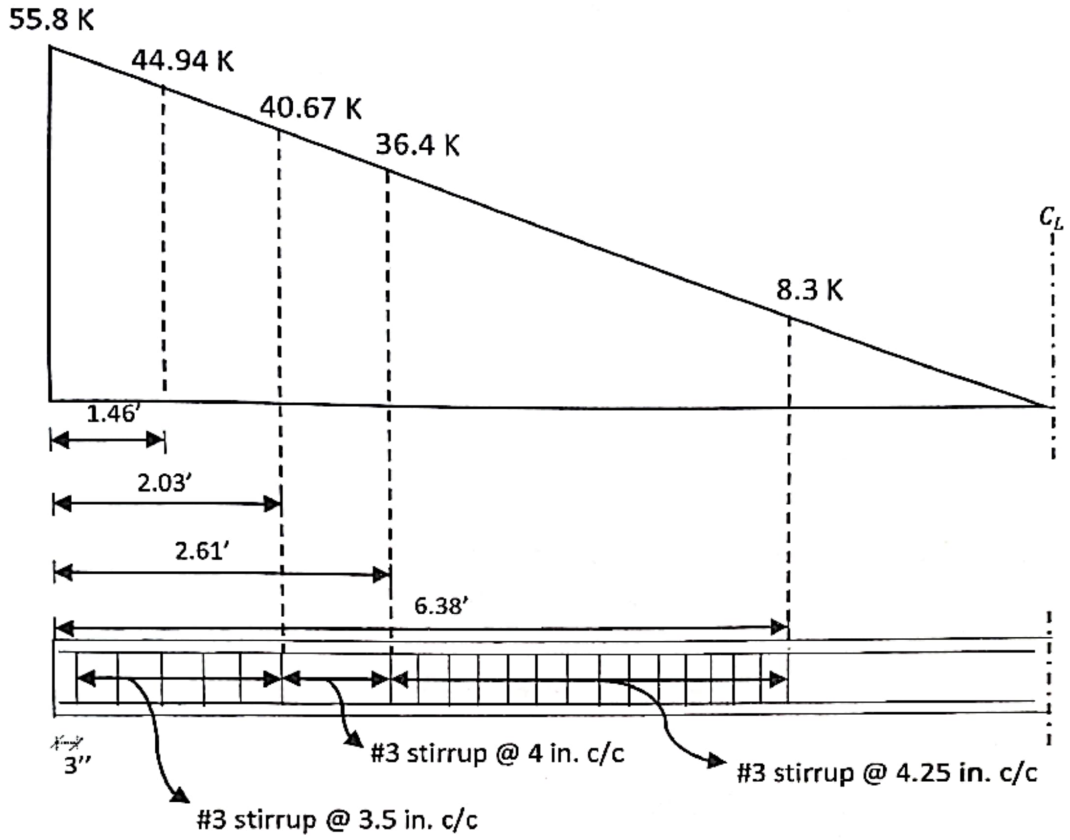
7. Spacing for critical shear V is,

$$S_1 = \frac{\varphi A_v f_{yt} d}{V_u - \varphi V_c} = \frac{0.75 \times (2 * 0.11) \times 60 \times 17.5}{44.94 - 16.6} = 6.1\ in. \approx 6\ in.$$

The first stirrup is placed at distance $\frac{S_1}{2} = \frac{6}{2} = 3\ in.$ from the face of the support.

[continue.....]

8. Reinforcement detailing:



9. Stirrup distribution:

i) 1 first stirrup @ 3 in. = 3 in.

ii) 4 stirrup @ 6 in. = 24 in.

iii) 1 stirrup @ 7 in. = 7 in.

iv) 5 stirrup @ 8.75 ≈ 8.5 in. = 42.5 in.

Total = 76.5 in. = 6.375 ft (≈6.38 ft)

Chapter-5
Bond, Anchorage and Development Length

5.1. Theory Analysis:

Q-1. Define bond strength. Why is it essential to provide between steel and concrete in beams?

Ans: ***Bond strength:*** Bond strength is defined as the shear force per unit nominal surface area of reinforcing bars. The bond strength is acting on the interface between bars and surrounding concrete and along the direction parallel to the bars.

Essentiality of bond strength: The bond strength between steel and concrete is very important and essential so that they can act together without any slip in a loaded structure. If there is no bonding between the two materials and if the bars are not anchored at their ends, they will pull loose from the concrete. As a result, the concrete beam will act as an unreinforced member and will be subject to sudden collapse as soon as the concrete cracks. With the perfect bond between them, the plane section of a beam remains plane even after bending.

Q-2. Describe how bond is maintained between steel and concrete in a RCC member. [***CE-2015***]

Ans: Bond maintenance between steel and concrete: Bond is maintained by the three process that is described below:

(i) Chemical adhesion: When the force is first applied to the bar, the resistance to slipping is provided by the chemical adhesion between the steel bar and the concrete.

(ii) Mechanical friction: If plain bars were used, it would not take much tension in the bars to break this adhesion, particularly adjacent to a crack in the concrete. If this were to happen for a smooth surface bar, only mechanical friction would remain to keep the bar from slipping.

(iii) Rib-shaped deformed bar: Deformed bars were introduced so that in addition to the adhesion and friction, there would also be a resistance due to the bearing of the concrete on the ribs of the bars as well as the so-called shear-friction strength of the concrete between the ribs.

Q-3. Describe the various types of bond failures with neat sketches. [***CE-2019, 2018, 2017, 2015***]
or, What are the common types of bond failures in reinforced concrete beam? What are the main reasons behind them? Show with neat sketch. [***CE-2013***]

Ans: The common types of bond failures in reinforced concrete beam are described below:
(i) Direct pullout or slip: When the bar is confined by surrounding concrete, then as the tensile force on the bar is increased, adhesive bond and friction are overcome, the concrete eventually crushes locally, which is called direct pullout.

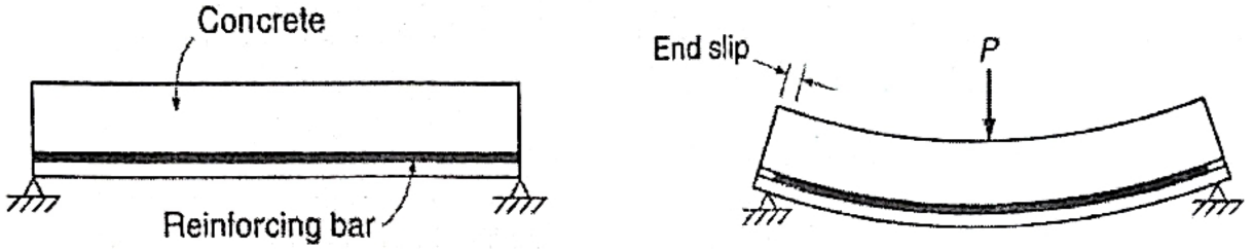


Fig: Direct pullout of the bar

(ii) Splitting: Bond failure resulting from splitting of the concrete is more common in beams than direct pullout. Splitting comes mainly from wedging action when the ribs of the deformed bars bear against the concrete. It may occur either in a vertical plane or horizontal plane of the bars.

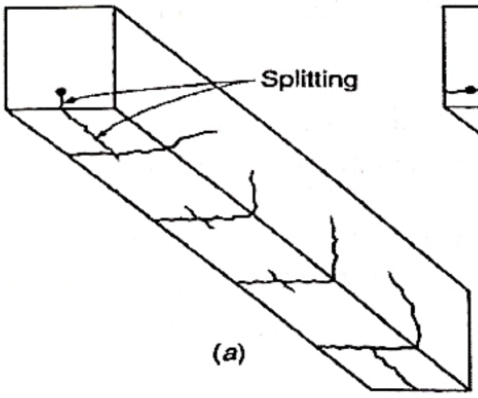


Fig: Vertical splitting

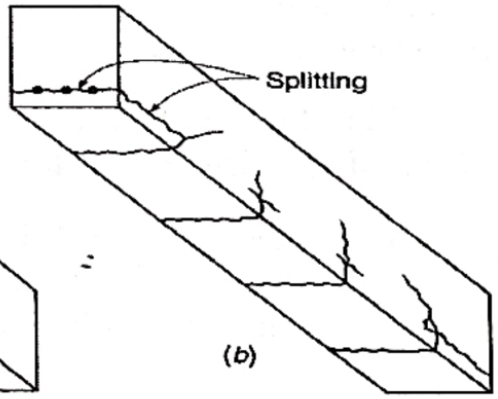


Fig: Horizontal splitting

Q-4. Define development length. Why is it necessary to maintain development length for the reinforcing bars? Explain with appropriate sketches. [***CE-2016, 2015***]
Ans: ***Development length:*** Development length is defined as the minimum length of the bar required to transfer stress from steel to concrete.

It is attained by extending the straight bar a sufficient distance from the point of maximum stress (M_{max} *or* σ_{max}).

Necessary to maintain development length: Development length is needed to establish the desired bond strength between the concrete and steel. Further, bond strength is required to form a continuous structure. The development length also resists concrete cracks by controlling the pullout and splitting. However, it is provided to have a fixed support to the bars to withhold the position of stressed bars in the concrete. So, it is greatly necessary to maintain development length for the reinforcing bars in the concrete.

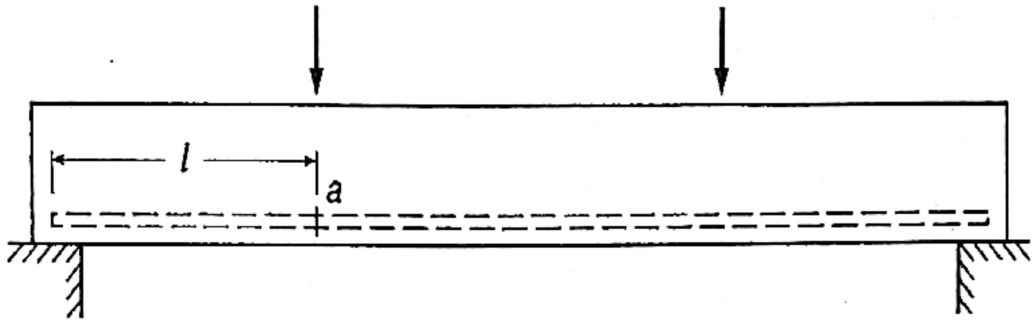


Fig: Development length

Q-5. What is the function of development length for reinforcing bars? [***CE-2013***]

Ans: The functions of development length for reinforcing bars are described below:

(i) It is provided to create a bond between steel and concrete in order to transfer the bar force without slip to prevent the bar from pullout.

(ii) It strengthens the reinforced concrete member by establishing desired bond strength.

(iii) It prevents local bond failures of pullout and splitting.

(iv) It resists sliding of reinforcing bars in the concrete.

(v) It provides the mechanical interlock of the deformed bars with the surrounding concrete.

Q-6. What are the factors of influencing of development length for reinforcing bars?

Ans: The factors of influencing of development length for reinforcing bars are described below:

(i) Location of reinforcement

(ii) Coating of bars

(iii) Size of reinforcement

(iv) Lightweight aggregates

(v) Spacing of bars

(vi) Cover dimensions

(vii) Transverse reinforcement (stirrups)
(viii) Grade of steel and concrete

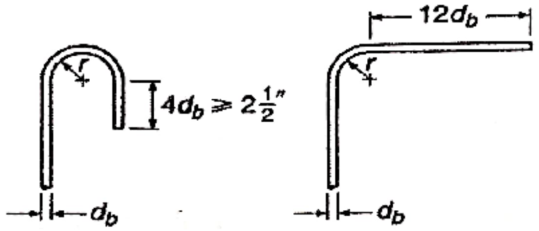
Q-7. Define hook? When hooks are required in reinforced concrete member? Show the standard hook dimensions according to ACI code by neat sketches.

[CE-2016, 2014; BECM-2018, 2017]

Ans: ***Hook:*** Hook is defined as the headed bar at the end of a bar when its straight embedment length is less than the necessary development length.

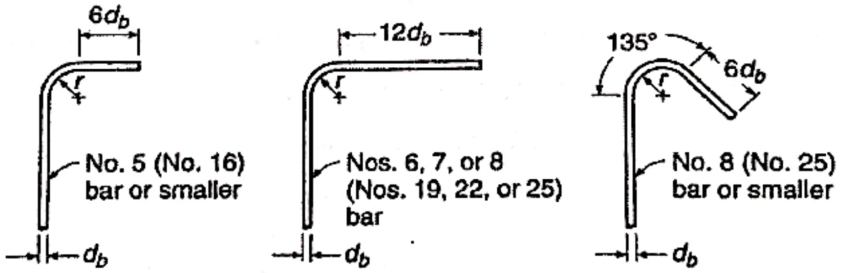
When hooks are required: When sufficient space is not available to anchor tension bars by running them straight for their required development lengths. Also tensile stress cannot be developed by bond alone, then it is necessary to provide special anchorage of hooks.

Standard hook dimensions according to ACI code:
(i) For main reinforcement:



(a) 180° hook (b) 90° hook

(i) For stirrups and ties:



Q-8. What is bar splicing? What are the provisions of ACI code for bar splicing in tension?
Ans: ***Bar splicing:*** When some reinforced bars are short in structural member then two reinforced bars of standard lengths are joined and are then wired or lapped together to form a single rebar that has higher length and it is called bar splicing.

The provisions of ACI code for bar splicing in tension:
The ACI code introduces two classes of splices depending upon the percentage of bars spliced on the same location and the level of stress in the bars.

(i) Class A splices: These splices have a minimum length $l_{st} = 1.0l_d$ are used when (a) one-half or less of the total reinforcement is spliced within the required lap length; and (b) the area of reinforcement provided is at least twice that required by analysis over the entire length of the splice.
(ii) Class B splices: These splices have a minimum length $l_{st} = 1.3l_d$ are used for all other cases that are different from the above conditions.

Q-9. Why bars are cut off in flexural member?
Ans: Four major factors affect the location of bar cutoffs.
(i) Bars can be cut off where they are no longer needed to resist tensile forces or where remaining bars are adequate to do so.
(ii) There must be sufficient extension of each bar, on each side of every section, to develop the force in that bar at that section.
(iii) Tension bars are cut off in a region of moderately high shear force cause a major stress concentration, which can lead to major inclined cracks at the bar cutoff.
(iv) Certain constructional requirements are specified in the cod as good practice.

Q-10. What are the effects of coating on reinforcement?
Ans: Reinforcement is usually coated to reduce the corrosion of reinforcement. From the basic equation we know, $l_d \propto \Psi_e$. So if the reinforcement is coated, it should be provided 1.2 or 1.5 times longer than before reinforcement. Although the project cost is high, the additional benefits are available.

Q-11. What are the differences among development length, anchorage length and lap length?
Ans: The differences among development length, anchorage length and lap length are described below:-

Development length	Anchorage length	Lap length
Development length is defined as the minimum length of the bar required to transfer stress from steel to concrete.	Anchorage length is the length which is provided if sufficient development length cannot be able to be provided inside support.	Lap length is the minimum length of the overlap bar required to transfer stress from one bar to another safely.
It is attained by extending the straight bar a sufficient distance from the point of maximum stress (M_{max} or σ_{max}).	It is attained by providing hook at the end of a bar.	It is attained by splicing of two bar side by side which gives required design length.

5.2. Basic Concepts:

1. Concrete aggregate factor, λ:

Aggregate type	Aggregate factor, λ
Normal-weight concrete	1.0
Sand, CA light-weight concrete	0.85
All light-weight concrete	0.75

2. Factor values for calculating l_d for bars in Tension:

Influencing factor		Factor value
Reinforcement location factor	Ψ_t	= 1.3 for top bars (having 12 in. fresh concrete below the bar).
		= 1.0 for other bars.
Coating factor	Ψ_e	= 1.0 for uncoated bars.
		= 1.2 for epoxy coated bars.
		= 1.5 for epoxy coated bars with cover.
Reinforcement size factor	Ψ_s	= 0.8 for #6 and smaller bars.
		= 1.0 for #7 and larger bars.

*In few textbooks, the influencing factor is symbolized as follows:

$$\Psi_t = \alpha$$

$$\Psi_e = \beta$$

$$\Psi_s = \gamma$$

5.3. Solution Techniques:

5.3.1 Development length of tension bars:

A. Basic equation:

Step-1. Cover dimension, c_b:

i) One-half the center to center spacing of bars being developed = √
ii) Distance from the bar center to the nearest concrete surface,

to the side = √
to the top = √
$\therefore c_b = minimum$

Step-2. Transverse reinforcement index, k_{tr}:
Failure of main bar, n = 2 [as per question]

$$\therefore k_{tr} = \frac{40A_{tr}}{Sn}$$

Step-3.

$$\therefore \frac{c_b + k_{tr}}{d_b} \leq 2.5$$

Step-4. Development length:
Factors, Ψ_t, Ψ_e, Ψ_s & λ

$$l_d = \frac{3}{40} \cdot \frac{f_y}{\lambda\sqrt{f_c'}} \cdot \frac{\Psi_t\Psi_e\Psi_s}{\left(\frac{c_b + k_{tr}}{d_b}\right)} * d_b$$

B. Simplified equation:

1. Check for simplified equation:

i) Clear spacing between bars = ☐ $\geq d_b$

ii) Clear cover over primary reinforcement,

to the side = ☐ $\geq d_b$

to the top or bottom = ☐ $\geq d_b$

iii) Stirrup criteria satisfied

2. Development length:

Factors, Ψ_t, Ψ_e, Ψ_s & λ

$$l_d = \frac{1}{20} \cdot \frac{f_y}{\lambda\sqrt{f_c'}} \cdot \Psi_t\Psi_e\Psi_s * d_b$$

5.3.2 Development length of compression bars:

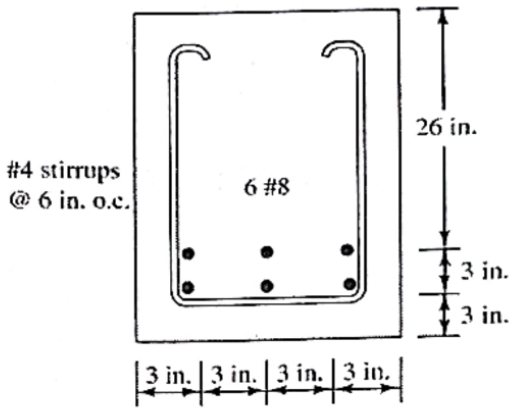
The development length in compression is given by the equation,

$$l_{dc} = \left(\frac{0.02f_y}{\lambda\sqrt{f_c'}}\right) d_b$$

$$or,\ l_{dc} = 0.0003\, f_y d_b > 8\ in.$$

5.4. Problem Solution:

Pb-1. Determine the development length required for the tension bars, as shown in the figure below i) by using basic equation and ii) by using simplified equation. Bars are uncoated in normal-weight concrete. Use $f_c' = 4000\ psi$ *and* $f_y = 60{,}000\ psi$.
[*CE-2019, 2018*]



Sol^n:

A. Basic equation:

1. Cover dimension, c_b:

i) One-half the center to center spacing of bars being developed $= \frac{1}{2} \times (3) = 1.5\ in.$

ii) Distance from the bar center to the nearest concrete surface,
to the side = 3 *in.*
to the top = 3 *in.*
$\therefore c_b = 1.5\ in.$

2. Transverse reinforcement index, k_{tr}:

Failure of main bar, n = 3

$$\therefore k_{tr} = \frac{40A_{tr}}{Sn} = \frac{40 \times (2 * 0.20)}{6 \times 3} = 0.889$$

3.

$$\therefore \frac{c_b + k_{tr}}{d_b} = \frac{1.5 + 0.889}{1.0} = 2.389 \leq 2.5 \quad (ok)$$

4. Development length:

Factors, $\Psi_t = 1.0$ for bottom bars
$\Psi_e = 1.0$ for uncoated bars
$\Psi_s = 1.0$ for #8 bars
& $\lambda = 1.0$ for normal-weight concrete

$$l_d = \frac{3}{40} \cdot \frac{f_y}{\lambda\sqrt{f_c'}} \cdot \frac{\Psi_t \Psi_e \Psi_s}{\left(\frac{c_b + k_{tr}}{d_b}\right)} * d_b$$

$$\therefore l_d = \frac{3}{40} \cdot \frac{60{,}000}{1.0 \times \sqrt{4000}} \cdot \frac{1.0 \times 1.0 \times 1.0}{2.389} * 1.0 = 29.78 \approx 30\ in.$$

B. Simplified equation:

1. Check for simplified equation:

i) Clear spacing between bars $= 3 - 1.0 = 2\ in. \geq d_b (1.0\ in.)$

ii) Clear cover over primary reinforcement,
to the side $= 3 - \frac{1.0}{2} = 2.5\ in. \geq d_b (1.0\ in.)$

to the bottom $= 3 - \frac{1.0}{2} = 2.5\ in. \geq d_b (1.0\ in.)$

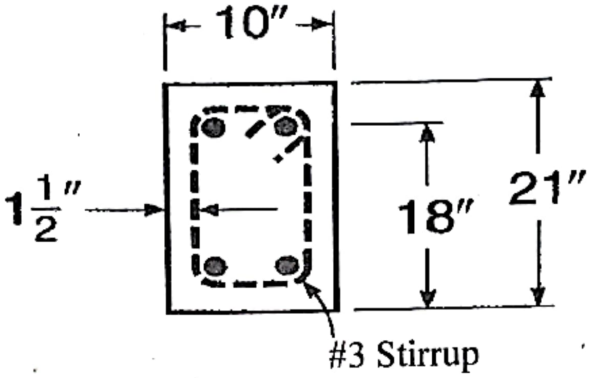
iii) Stirrup criteria satisfied

2. Development length:

$$l_d = \frac{1}{20} \cdot \frac{f_y}{\lambda\sqrt{f_c'}} \cdot \Psi_t \Psi_e \Psi_s * d_b$$

$$\therefore l_d = \frac{1}{20} \cdot \frac{60{,}000}{1.0 \times \sqrt{4000}} \cdot 1.0 \times 1.0 \times 1.0 * 1.0 = 47.43\ in.$$

Pb-2. In a beam-column joint of a continuous building frame, based on frame analysis, the negative steel required at the end of the beam is 2.90 in^2, two No. 11 bars are used, providing 3.12 in^2. Beam dimensions are b = 10 in., d = 18 in. and h = 21 in. The design will include No. 3 stirrups at 5 in. spacing, with 1.5 in. clear cover. Normal-weight concrete is to be used, with $f_c' = 4000\ psi\ and\ f_y = 60{,}000\ psi$. Find the minimum development length of the required steel area at the face of the column, (i) using the basic equation and (ii) using simplified equations.



Sol^n:

A. <u>Basic equation:</u>

1. Cover dimension, c_b:

i) One-half the center to center spacing of bars being developed

$$= \frac{1}{2} \times (10 - 2 * 1.5 - 2 * 0.38 - 1.41) = 2.42\ in.$$

ii) Distance from the bar center to the nearest concrete surface,

to the side $= \frac{1.41}{2} + 0.38 + 1.5 = 2.58\ in.$

to the top = 3 $in.$

$\therefore c_b = 2.42\ in.$

2. Transverse reinforcement index, k_{tr}:

Failure of main bar, n = 2

$$\therefore k_{tr} = \frac{40A_{tr}}{Sn} = \frac{40 \times (2 * 0.11)}{5 \times 2} = 0.88$$

3.

$$\therefore \frac{c_b + k_{tr}}{d_b} = \frac{2.42 + 0.88}{1.41} = 2.34 \leq 2.5 \quad (ok)$$

4. Development length:

Factors, $\Psi_t = 1.3$ for top bars

$\Psi_e = 1.0$ for uncoated bars

$\Psi_s = 1.0$ for #8 bars

& $\lambda = 1.0$ for normal-weight concrete

$$l_d = \frac{3}{40}.\frac{f_y}{\lambda\sqrt{f_c'}}.\frac{\Psi_t\Psi_e\Psi_s}{\left(\frac{c_b + k_{tr}}{d_b}\right)} * d_b$$

$$\therefore l_d = \frac{3}{40}.\frac{60{,}000}{1.0 \times \sqrt{4000}}.\frac{1.3 \times 1.0 \times 1.0}{2.34} * 1.41 = 55.7\ in.$$

$$Revised\ l_d = l_d * \frac{A_s(required)}{A_s(provided)} = 55.7 * \frac{2.90}{3.12} = 51.8 \approx 52\ in.$$

B. Simplified equation:

1. Check for simplified equation:

i) Clear spacing between bars $= 10 - 2 * 1.5 - 2 * 0.38 - 2 * 1.41 = 3.42\ in. \geq d_b(1.41\ in.)$

ii) Clear cover over primary reinforcement,
to the side = c.c + Dia. of stirrup $= 1.5 + 0.38 = 1.88\ in. \geq d_b(1.41\ in.)$

to the top $= 3 - \frac{1.41}{2} = 2.29\ in. \geq d_b(1.41\ in.)$

iii) Stirrup criteria satisfied

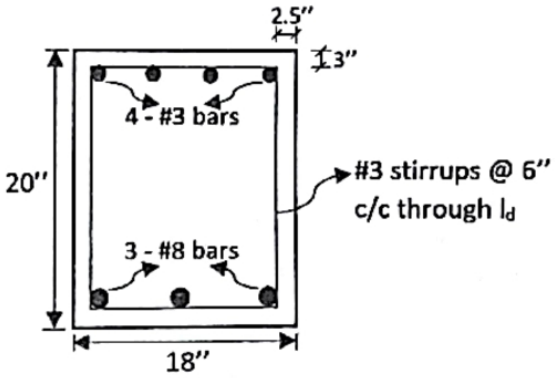
2. Development length:

$$l_d = \frac{1}{20}.\frac{f_y}{\lambda\sqrt{f_c'}}.\Psi_t\Psi_e\Psi_s * d_b$$

$$\therefore l_d = \frac{1}{20}.\frac{60{,}000}{1.0 \times \sqrt{4000}}.1.3 \times 1.0 \times 1.0 * 1.41 = 87\ in.$$

$$Revised\ l_d = l_d * \frac{A_s(required)}{A_s(provided)} = 87 * \frac{2.90}{3.12} = 81\ in.$$

Pb-3. Figure shows the cross-section of a rectangular beam near beam-column joint. Compute the development length for the negative reinforcement.
Use $f_c' = 3000\ psi\ and\ f_y = 60{,}000\ psi$. [*CE-2017*]



Sol^n:

A. <u>Basic equation:</u>

1. <u>Cover dimension, c_b:</u>

i) One-half the center to center spacing of bars being developed

$$= \frac{1}{2} \times (18 - 2 * 2.5 - 2 * 0.38 - 3 * 0.38)$$

$$= 5.55\ in.$$

ii) Distance from the bar center to the nearest concrete surface,

to the side $= \frac{0.38}{2} + 0.38 + 2.5 = 3.07\ in.$

to the top $= \frac{0.38}{2} + 0.38 + 3 = 3.57\ in.$

$\therefore c_b = 3.07\ in.$

2. Transverse reinforcement index, k_{tr}:

Failure of main bar, n = 4

$$\therefore k_{tr} = \frac{40A_{tr}}{Sn} = \frac{40 \times (2 * 0.11)}{6 \times 4} = 0.367$$

3.

$$\therefore \frac{c_b + k_{tr}}{d_b} = \frac{3.07 + 0.143}{0.38} = 9.045 \leq 2.5 \approx 2.5$$

4. <u>Development length:</u>

Factors, $\Psi_t = 1.3$ for top bars

$\Psi_e = 1.0$ for uncoated bars

$\Psi_s = 0.8$ for #3 bars

& $\lambda = 1.0$ for normal-weight concrete

$$l_d = \frac{3}{40}.\frac{f_y}{\lambda\sqrt{f_c'}}.\frac{\Psi_t\Psi_e\Psi_s}{\left(\frac{c_b + k_{tr}}{d_b}\right)} * d_b$$

$$\therefore l_d = \frac{3}{40}.\frac{60{,}000}{1.0 \times \sqrt{3000}}.\frac{1.3 \times 1.0 \times 0.8}{2.5} * 0.38 = 12.98\ in. \approx 13\ in.$$

B. Simplified equation:

1. Check for simplified equation:

i) Clear spacing between bars $= 18 - 2 * 2.5 - 2 * 0.38 - 4 * 0.38$

$= 10.72\ in. \geq d_b(0.38\ in.)$

ii) Clear cover over primary reinforcement,
to the side $= 2.5 + 0.38 = 2.88\ in. \geq d_b(0.38\ in.)$

to the top $= 3 + 0.38 = 3.38\ in. \geq d_b(0.38\ in.)$

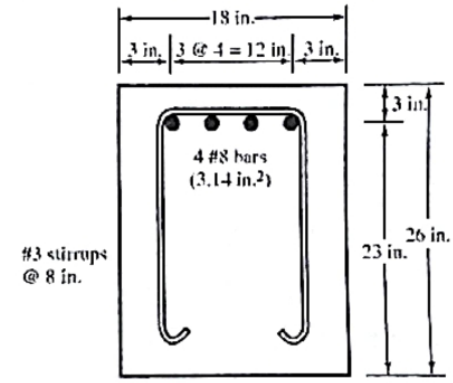
iii) Stirrup criteria satisfied

2. Development length:

$$l_d = \frac{1}{20}.\frac{f_y}{\lambda\sqrt{f_c'}}.\Psi_t\Psi_e\Psi_s * d_b$$

$$\therefore l_d = \frac{1}{20}.\frac{60{,}000}{1.0 \times \sqrt{3000}}.1.3 \times 1.0 \times 0.8 * 0.38 = 21.65\ in. \approx 22\ in.$$

Pb-4. The required reinforcing steel area for the lightweight concrete beam in figure below is 2.88 in^2. The #8 top bars shown are uncoated. Compute the development length for the following cases, if $f_c{}' = 3500\ psi\ and\ f_y = 60{,}000\ psi$. [***CE-2015; BECM-2018***]



Sol^n:

A. Basic equation:

1. Cover dimension, c_b:

i) One-half the center to center spacing of bars being developed

$$= \frac{1}{2} \times (4) = 2\ in.$$

ii) Distance from the bar center to the nearest concrete surface,
to the side = 3 $in.$
to the top = 3 $in.$

$$\therefore c_b = 2\ in.$$

2. Transverse reinforcement index, k_{tr}:

Failure of main bar, n = 4

$$\therefore k_{tr} = \frac{40A_{tr}}{Sn} = \frac{40 \times (2 * 0.11)}{8 \times 4} = 0.275$$

3.

$$\therefore \frac{c_b + k_{tr}}{d_b} = \frac{2 + 0.275}{1.0} = 2.275 \leq 2.5 \quad (ok)$$

4. Development length:

Factors, $\Psi_t = 1.3$ for top bars

$\Psi_e = 1.0$ for uncoated bars

$\Psi_s = 1.0$ for #8 bars

& $\lambda = 0.75$ for lightweight concrete

$$l_d = \frac{3}{40} \cdot \frac{f_y}{\lambda\sqrt{f_c{}'}} \cdot \frac{\Psi_t \Psi_e \Psi_s}{\left(\frac{c_b + k_{tr}}{d_b}\right)} * d_b$$

$$\therefore l_d = \frac{3}{40} \cdot \frac{60{,}000}{0.75 \times \sqrt{3500}} \cdot \frac{1.3 \times 1.0 \times 1.0}{2.275} * 1.0 = 57.95\ in.$$

$$Revised\ l_d = l_d * \frac{A_s(required)}{A_s(provided)} = 57.95 * \frac{2.88}{3.16} = 52.82 \approx 53\ in.$$

B. Simplified equation:

1. Check for simplified equation:

i) Clear spacing between bars $= 4 - 1.0 = 3\ in. \geq d_b (1.0\ in.)$

ii) Clear cover over primary reinforcement,

to the side $= 3 - \frac{1.0}{2} = 2.5\ in. \geq d_b (1.0\ in.)$

to the top $= 3 - \frac{1.0}{2} = 2.5\ in. \geq d_b (1.0\ in.)$

iii) Stirrup criteria satisfied

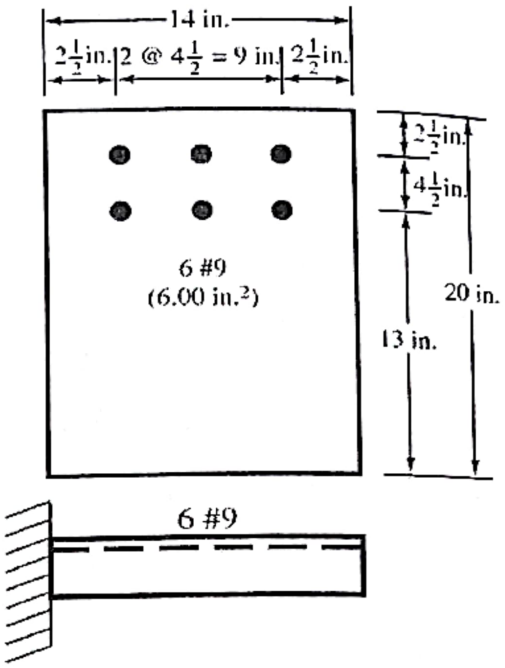
2. Development length:

$$l_d = \frac{1}{20} \cdot \frac{f_y}{\lambda \sqrt{f_c'}} \cdot \Psi_t \Psi_e \Psi_s * d_b$$

$$\therefore l_d = \frac{1}{20} \cdot \frac{60{,}000}{0.75 \times \sqrt{3500}} \cdot 1.3 \times 1.0 \times 1.0 * 1.0 = 87.90\ in.$$

$$Revised\ l_d = l_d * \frac{A_s(required)}{A_s(provided)} = 87.90 * \frac{2.88}{3.16} = 81\ in.$$

Pb-5. Determine the development length required for the epoxy-coated bars of the beam as shown in figure below, for the following cases
(i) If bars are straight.
(ii) If a 180° hook is used.
(iii) If a 90° hook is used.
The six #9 bars shown are considered to be top bars.
Use $f_c' = 3500\ psi\ and\ f_y = 60{,}000\ psi.$
[*CE-2014*]



Sol^n:

(i) If bars are straight:

Using basic equation:

1. Cover dimension, c_b:

i) One-half the center to center spacing of bars being developed

$$= \frac{1}{2} \times (4.5) = 2.25\ in.$$

ii) Distance from the bar center to the nearest concrete surface,
to the side = 2.5 *in.*
to the top = 2.5 *in.*

$\therefore c_b = 2.25\ in.$

2. Transverse reinforcement index, k_{tr}: Assuming, $k_{tr} = 0$

3.

$$\therefore \frac{c_b + k_{tr}}{d_b} = \frac{2.25 + 0}{1.128} = 1.99 \leq 2.5 \quad (ok)$$

4. Development length:

Factors, $\Psi_t = 1.3$ for top bars

$\Psi_e = 1.2$ for coated bars

$\Psi_s = 1.0$ for #9 bars

& $\lambda = 1.0$ for normal-weight concrete

$\therefore \Psi_t\Psi_e = 1.3 * 1.2 = 1.56 \leq 1.7 \quad (ok)$

$$Now,\ l_d = \frac{3}{40}.\frac{f_y}{\lambda\sqrt{f_c'}}.\frac{\Psi_t\Psi_e\Psi_s}{\left(\frac{c_b + k_{tr}}{d_b}\right)} * d_b$$

$$\therefore l_d = \frac{3}{40}.\frac{60{,}000}{1.0 \times \sqrt{4000}}.\frac{1.56 \times 1.0}{1.99} * 1.128 = 63\ in.$$

(ii) If a 180° hook is used:

Here, $\Psi_e = 1.2$ for epoxy-coated hooks.

$$l_{dh} = \frac{0.02 f_y \Psi_e}{\lambda\sqrt{f_c'}}.d_b = \frac{0.02 \times 60{,}000 \times 1.2}{1.0\sqrt{4000}} * 1.128 = 25.68 \approx 26\ in.$$

(iii) If a 90° hook is used:

Bars with 90° hooks has the same development length, as calculated before. No other modifications apply, then

$$l_{dh} = \frac{0.02 f_y \Psi_e}{\lambda\sqrt{f_c'}}.d_b = \frac{0.02 \times 60{,}000 \times 1.2}{1.0\sqrt{4000}} * 1.128 = 25.68 \approx 26\ in.$$

Pb-6. A reinforced concrete column is reinforced with eight No. 10 bars, which should extend to the footing. Determine the development length needed for the bars to extend down in the footing. Use normal-weight concrete with $f_c' = 4000\ psi$ *and* $f_y = 60{,}000\ psi$.

Sol^n:

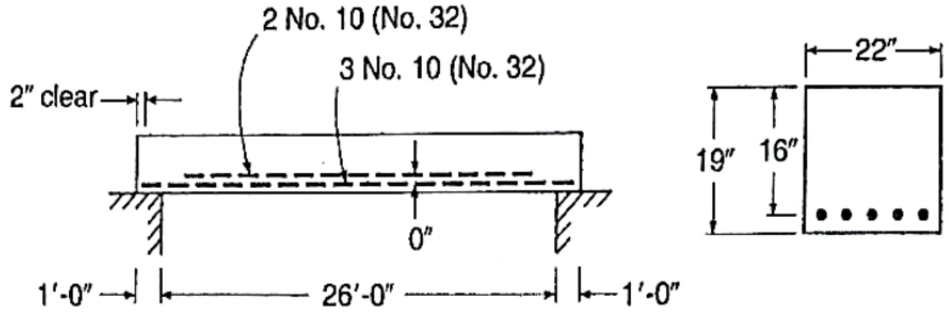
The development length in compression is,

$$l_{dc} = \left(\frac{0.02 f_y}{\lambda\sqrt{f_c'}}\right) d_b > 8\ in.$$

$$\Rightarrow l_{dc} = \left(\frac{0.02 \times 60000}{1.0\sqrt{4000}}\right) \times 1.27$$

$$\therefore l_{dc} = 24.1\ in. > 8\ in. \qquad (ok)$$

Pb-7. The beam shown in figure below is reinforced with 5 - #10 bars at en effective depth of 16 in. Two of the bars are to be discontinued where no longer needed. No. 3 stirrups are used at 6 in. uniform spacing throughout the beam with a cover of 1.5 in. in all sides. Use $f_c' = 4000\ psi\ and\ f_y = 60{,}000\ psi$. Calculate the point where those two bars can be discontinued. [***CE-2016***]



Sol^n:

Self-weight $= \frac{22 \times 19}{144} \times 150 \times 1.2 = 522.5\ plf = 0.52\ k/ft$

Live load (assume 2 kips/ft.) $= 2 \times 1.6 = 3.2\ k/ft$

Total load, $w = 3.72\ k/ft$

Reaction in support, $R = \frac{wl}{2} = \frac{3.72 \times 26}{2} = 48.36\ kips$

When two bars can be discontinued,

$$\therefore a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2 * 1.27 \times 60}{0.85 \times 4 \times 22} = 2.04$$

$$M_u = \varphi A_s f_y \left(d - \frac{a}{2}\right)$$

$$\therefore M_u = 0.90 \times 2 * 1.27 \times 60 \left(16 - \frac{2.04}{2}\right) = 2054.66\ k - in. = 171.22\ k - ft$$

Now, the moment at any section in the beam at a distance x from the left support is as follows,

$$M = Rx - \frac{wx^2}{2}$$

$$\Rightarrow 171.22 = 48.36x - \frac{3.72x^2}{2}$$

$$\Rightarrow 171.22 = 48.36x - 1.86x^2$$

$$\Rightarrow 1.86x^2 - 48.36x + 171.22 = 0$$

$$\therefore x = 4.23,\ 21.77$$

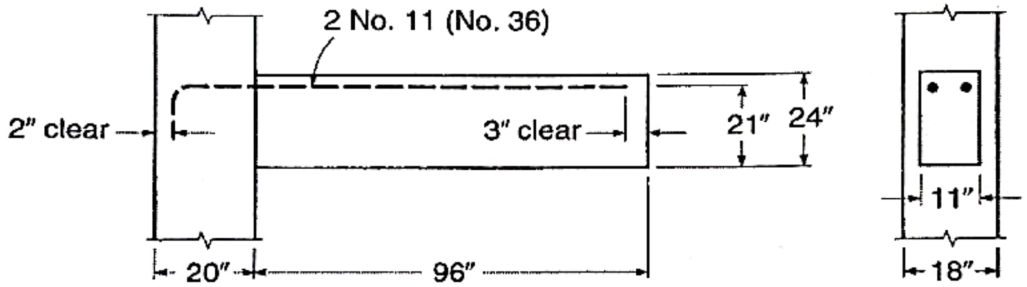
$\because x = 4.23$ ft. from the left support.

The two discontinued bars must be continued at least $d = 16'' = 1.33$ ft. beyond this theoretical point of cutoff.

$\therefore$ Actual cutoff or discontinued point $= 4.23 - 1.33 = 2.90$ ft. from the both support.

(Ans)

Pb-8. A cantilever beam carries a calculated dead load of 2.0 kips/ft. including its self-weight and a service live load of 2.6 kips/ft. Tensile flexural reinforcement consists of two No. 11 bar at a 21 in. effective depth. Transverse No. 3 U-stirrups with 1.5 in. cover are provided at the following spacings from the face of the column: 4 in., 3 at 8 in., 5 at 10.5 in. If the beam uses light-weight concrete having $f_c' = 4000\ psi$ *and* $f_y = 60{,}000\ psi$. Check to see if proper development length can be provided for the No. 11 bars. [***CE-2013***]



Sol^n:

Dead load $= 1.2 \times 2.0 = 2.4\ k/ft$

Live load $= 1.6 \times 2.6 = 4.16\ k/ft$

Total load, $w = 6.56\ k/ft$

$$Moment,\ M = \frac{wl^2}{2} = \frac{6.56 \times \left(\frac{96}{12}\right)^2}{2} = 210\ k - ft$$

$$Here,\ a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2 * 1.56 \times 60}{0.85 \times 4 \times 11} = 5$$

$$A_s(required) = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} = \frac{210 \times 12}{0.90 \times 60\left(21 - \frac{5}{2}\right)} = 2.52\ in^2$$

$$A_s(provided) = 2 * 1.56 = 3.12\ in^2$$

Using simplified equation:

1. Check for simplified equation:

i) Clear spacing between bars $= 11 - 2 * 1.5 - 2 * 0.38 - 2 * 1.41$

$$= 4.42\ in. \geq d_b(1.41\ in.)$$

ii) Clear cover over primary reinforcement,
to the side $= 1.5 + 0.38 = 1.88\ in. \geq d_b(1.41\ in.)$

to the top $= 3 + 0.38 = 3.38\ in. \geq d_b(1.41\ in.)$

iii) Stirrup criteria satisfied

2. Development length:

Factors, $\Psi_t = 1.3$ for top bars

$\Psi_e = 1.0$ for uncoated bars

$\Psi_s = 1.0$ for #11 bars

& $\lambda = 0.75$ for lightweight concrete

$$l_d = \frac{1}{20}.\frac{f_y}{\lambda\sqrt{f_c'}}.\Psi_t\Psi_e\Psi_s * d_b$$

$$\therefore l_d = \frac{1}{20}.\frac{60{,}000}{0.75 \times \sqrt{4000}}.1.3 \times 1.0 \times 1.0 * 1.41 = 115.93\ in.$$

$$Revised\ l_d = l_d * \frac{A_s(required)}{A_s(provided)} = 115.93 * \frac{2.52}{3.12} = 93.64 \approx 94\ in.$$

$$Provided\ l_d = 20 + 96 - 2 - 3 = 111\ in.$$

$$Provided\ l_d > Revised\ l_d$$

[Checked & ok]

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3. Design of Reinforced Concrete (9th edition)
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6. Building Code Requirements for Structural Concrete (ACI 318-08)

Appendix A: Necessary Tables

Table A.1: Designation, diameter and area of standard bars

Bar size		Diameter (in.)	Area (in^2)
Bar No.	**Bar (mm)**		
3	10	0.375	0.11
4	13	0.5	0.20
5	16	0.625	0.31
6	19	0.75	0.44
7	22	0.875	0.60
8	25	1.0	0.79
9	29	1.128	1.00
10	32	1.27	1.27
11	36	1.41	1.56
14	43	1.693	2.25
18	57	2.25	4.00

*[For finding bar diameter from **bar no. to mm (#3 to 10 mm)** & area by short techniques, follow **Appendix B** (Table B.1)]*

Table A.2: ACI Coefficients for Design of two-way slab

A.2.1: Coefficients for negative moments in slabs

$M_{a(-ve)} = C_{a(-ve)} * wl_a^2$; where, w = total uniform dead plus live load (DL+LL)

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,neg}$		0.045		0.050	0.075	0.071		0.033	0.061
	$C_{b,neg}$		0.045	0.076	0.050			0.071	0.061	0.033
0.95	$C_{a,neg}$		0.050		0.055	0.079	0.075		0.038	0.065
	$C_{b,neg}$		0.041	0.072	0.045			0.067	0.056	0.029
0.90	$C_{a,neg}$		0.055		0.060	0.080	0.079		0.043	0.068
	$C_{b,neg}$		0.037	0.070	0.040			0.062	0.052	0.025
0.85	$C_{a,neg}$		0.060		0.066	0.082	0.083		0.049	0.072
	$C_{b,neg}$		0.031	0.065	0.034			0.057	0.046	0.021
0.80	$C_{a,neg}$		0.065		0.071	0.083	0.086		0.055	0.075
	$C_{b,neg}$		0.027	0.061	0.029			0.051	0.041	0.017
0.75	$C_{a,neg}$		0.069		0.076	0.085	0.088		0.061	0.078
	$C_{b,neg}$		0.022	0.056	0.024			0.044	0.036	0.014
0.70	$C_{a,neg}$		0.074		0.081	0.086	0.091		0.068	0.081
	$C_{b,neg}$		0.017	0.050	0.019			0.038	0.029	0.011
0.65	$C_{a,neg}$		0.077		0.085	0.087	0.093		0.074	0.083
	$C_{b,neg}$		0.014	0.043	0.015			0.031	0.024	0.008
0.60	$C_{a,neg}$		0.081		0.089	0.088	0.095		0.080	0.085
	$C_{b,neg}$		0.010	0.035	0.011			0.024	0.018	0.006
0.55	$C_{a,neg}$		0.084		0.092	0.089	0.096		0.085	0.086
	$C_{b,neg}$		0.007	0.028	0.008			0.019	0.014	0.005
0.50	$C_{a,neg}$		0.086		0.094	0.090	0.097		0.089	0.088
	$C_{b,neg}$		0.006	0.022	0.006			0.014	0.010	0.003

† A crosshatched edge indicates that the slab continues across, or fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

A.2.2: Coefficients for dead load positive moments in slabs

$$M_{a(+ve)DL} = C_{a(+ve)DL} * wl_a^{\,2}$$

$$M_{b(+ve)DL} = C_{a(+ve)DL} * wl_a^{\,2} \quad ; \text{where, } w = \text{total dead load}$$

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,dl}$	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023
	$C_{b,dl}$	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020
0.95	$C_{a,dl}$	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
	$C_{b,dl}$	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
0.90	$C_{a,dl}$	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
	$C_{b,dl}$	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.85	$C_{a,dl}$	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
	$C_{b,dl}$	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.80	$C_{a,dl}$	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
	$C_{b,dl}$	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.75	$C_{a,dl}$	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
	$C_{b,dl}$	0.019	0.009	0.018	0.013	0.007	0.012	0.020	0.013	0.007
0.70	$C_{a,dl}$	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.033
	$C_{b,dl}$	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.65	$C_{a,dl}$	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.034
	$C_{b,dl}$	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.60	$C_{a,dl}$	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.036
	$C_{b,dl}$	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.55	$C_{a,dl}$	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.037
	$C_{b,dl}$	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.50	$C_{a,dl}$	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.038
	$C_{b,dl}$	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002

† A crosshatched edge indicates that the slab continues across, or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

A.2.3: Coefficients for live load positive moments in slabs

$M_{a(+ve)LL} = C_{a(+ve)LL} * wl_a{}^2$

$M_{b(+ve)LL} = C_{a(+ve)LL} * wl_a{}^2$; where, w = total live load

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,ll}$	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030
	$C_{b,ll}$	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.028
0.95	$C_{a,ll}$	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
	$C_{b,ll}$	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
0.90	$C_{a,ll}$	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
	$C_{b,ll}$	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.85	$C_{a,ll}$	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
	$C_{b,ll}$	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.80	$C_{a,ll}$	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
	$C_{b,ll}$	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.75	$C_{a,ll}$	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
	$C_{b,ll}$	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.70	$C_{a,ll}$	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
	$C_{b,ll}$	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.65	$C_{a,ll}$	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
	$C_{b,ll}$	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.60	$C_{a,ll}$	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
	$C_{b,ll}$	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.55	$C_{a,ll}$	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	00.063
	$C_{b,ll}$	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.50	$C_{a,ll}$	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
	$C_{b,ll}$	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.004

† A crosshatched edge indicates that the slab continues across, or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

Appendix B: Special Short Techniques

Table B.1: Determination of diameter and area of bars

Bar dia, $d_b = \frac{Bar\ No.}{8}\ (in.)$	*e.g;* Dia. of #3 bar, $d_b = \frac{3}{8} = 0.375\ in.$
Bar dia, $d_b = \frac{Bar\ No.}{8} * 25.4\ (mm)$	*e.g;* Dia. of #3 bar, $d_b = \frac{3}{8} * 25.4 = 9.53 \approx 10\ mm$

Table B.2: When RCC member is to be designed in *WSD* and when in *USD*

When in WSD	***When in WSD***
1. Directly *WSD* is said.	1. Directly *USD*/Strength Design Method is said.
2. To find allowable working moment.	2. To find factored moment/moment strength.
3. Given, f_s & f_c value.	3. Given, f_y & f_c' value.
4. Given, allowable working moment.	4. Given, factored moment.
	5. Given, service live load.

Table B.3: Strength Reduction Factor of RCC Structures

RCC Member	**Strength Reduction Factor, φ**
Beam for flexure	0.90
Beam for shear	0.75
Tied Column	0.65
Spiral Column	0.75

Table B.4: Clear Cover & Effective Cover of RCC Structures

RCC Structures	Clear Cover (cc)	Effective Cover $(ec) = cc + \frac{d_b}{2}$
Slab	0.75"	1"
Beam	2"	2.5"
Column	2"	2.5"
Foundation	3"	3.5"
Staircase	0.75"	1"
Lintel	1"	1.5"
Sunshade	0.75"	1"

Table B.5: Using of Reinforcement Bar in RCC Structures (normally used)

RCC Structures	Bar Size (No.)	Bar Size (mm)
Beam	#6, #7, #8, #9, #10	19, 22, 25, 29, 32 mm
Slab	#3, #4	10, 13 mm
Column	#5, #6, #7, #8	16, 19, 22, 25 mm
Wall Footing	#3, #4	10, 13 mm
Foundation	#7, #8, #9	22, 25, 29 mm
Staircase	#3, #4, #5	10, 13, 16 mm
Lintel	#3, #4, #5	10, 13, 16 mm
Sunshade	#3, #4	10, 13 mm

Appendix C: Necessary Unit Conversion

Table C.1: Necessary Unit Conversion

Length	**Pressure**
1 in. = 25.4 mm 1 m = 39.37 in. 1 m = 3.28 ft 1 m = 1.094 yd 1 mile = 1.61 km	1 kN/m^2 = 20.885 psf 1 MPa = 1 N/mm^2 = 145 psi 1 MPa = 10.197 kg/cm^2 1 tsf = 95.76 kN/m^2 (kPa) 1 bar = 14.5 psi
Area	**Force**
1 m^2 = 10.76 sft 1 acre = 4046.86 m^2 1 acre = 4840 yd^2 1 Dec = 435.6 sft 1 Katha = 720 sft	1 kN = 224.8 lb 1 ton = 2000 lb = 2 kips
Mass	**Moment**
1 kg = 2.2 lb 1 lb = 453 gm 1 ounce = 28.35 gm	1 kN. m = 0.737 kip − ft
Volume	**Unit Weight**
1 US gallon = 3.78 litre 1 m^3 = 100 litre 1 cft = 28.32 litre 1 m^3 = 35.31 cft	Steel: 488 lb/ft^3 = 78 kN/m^3 = 7850 kg/m^3 Concrete: 150 lb/ft^3 = 24 kN/m^3 = 2400 kg/m^3

Notes

Assalamuyalaikum.
Dear audience,
Hopefully, this book will be considered much more useful to you. We have always tried our best to keep the book completely flawless. However, if any mistake is caught in someone's eyes, I will be grateful if you inform me according to the following rules and we will correct it in the next edition. InshAllah.

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E-mail or whatsapp message will be sent according to the following rules:

Book name, Page No, Question No (Theory/Problem), What's error, Correction.

Ex: AHRCC-I, Page-28, Question-3 (Problem), $n = 7$, $n = 9$.

Any suggestions on quality improvement except error will be sincerely accepted.